Form Factors and Structure Functions

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Physics 226, Fall 2010
Running of Coupling Constants

Generic property of any field theory: higher order corrections (loops) induce momentum (distance) dependence of coupling constants.

This is known as an effect of “vacuum polarization”

The magnitude and the direction of the change depends on the type of the interaction

\[ \alpha_s(Q) = \frac{\hbar}{\Delta x} \]
Running of Coupling Constants

- QED (electromagnetic interactions): photons have no charge, so to first order include only fermion loops.

- Coupling constant depends on momentum transfer $Q^2 \sim \hbar^2/\Delta x^2$, the number of active fermions $n_f$ (with mass $(2m_f)^2 < Q^2$), and the value of the coupling measured at some physical scale $\mu$ (typically, an electron mass).

$$\alpha(Q^2) = \frac{\alpha(\mu^2)}{1 - n_f \frac{\alpha(\mu^2)}{3\pi} \log \left(\frac{Q^2}{\mu^2}\right)}$$
Fine Structure Constant

Low energies:

\[ \alpha^{-1} = 137.035999084(51) \]

High Energies:

\[ \hat{\alpha}(M_Z^2)^{-1} = 127.916(15) \]

![Graph showing measurements of \( \alpha^{-1}(Q^2) \) for \( Q^2 < 0 \). The results of the small-angle and large-angle Bhabha scattering measurements are shown as a solid circle and square, respectively. The corresponding reference \( Q^2 \) values at which the value of \( \alpha^{-1}(Q^2) \) is fixed to its expected value are shown as open symbols. The error bar on the large-angle point indicates the experimental and the total uncertainty.](image-url)
Strong Interactions

• Both quarks and gluons contribute to loops
  - Boson and fermion loops have opposite signs (statistics)
  - There exists a value of momentum \( \Lambda \sim 300 \text{ MeV} \) where the coupling constant is \( \sim \) infinite
    - Perturbative expansion breaks down

\[
\alpha_S(Q^2) = \frac{12\pi}{(33 - 2n_f) \log(Q^2/\Lambda^2)}
\]

- \( \alpha(1 \text{ GeV}^2) \approx 0.5 \)
- \( \alpha(M_Z^2) = 0.1184 \pm 0.0007 \)
The central value is determined as the weighted average of the individual measurements. For the error an overall, a-priori unknown, correlation coefficient is introduced and determined by requiring that the total $\chi^2$ of the combination equals the number of degrees of freedom. The world average quoted in Ref. 172 is $\alpha_s(M_Z) = 0.1184 \pm 0.0007$, with an astonishing precision of 0.6%. It is worth noting that a cross check performed in Ref. 172, consisting in excluding each of the single measurements from the combination, resulted in variations of the central value well below the quoted uncertainty, and in a maximal increase of the combined error up to 0.0012. Most notably, excluding the most precise determination from lattice QCD gives only a marginally different average value. Nevertheless, there remains an apparent and long-standing systematic difference between the results from structure functions and other determinations of similar accuracy. This is evidenced in Fig. 9.2 (left), where the various inputs to this combination, evolved to the $Z$ mass scale, are shown. Fig. 9.2 (right) provides strongest evidence for the correct prediction by QCD of the scale dependence of the strong coupling.

Two regimes: quarks are asymptotically free at short distances (high energies), but are confined to hadrons when pulled apart (low energies).
• **Goal:** quantitative understanding of hadronic structure
  - How does proton mass arise from its constituents (quarks and gluons)
  - How does proton spin arise from its constituents?

• **Complications:**
  - Strong interactions, quark confinement
  - **Main tool:** lepton (electron, muon) scattering
    - High energy probe, no strong interactions between target and...
Accelerator Probes of Matter

• “A better magnifying glass”
  - Need to look deeper into the structure of matter
  - Spatial resolution is limited by wavelength of the probe
    - Microscope: visible light $\lambda \sim 1 \mu m \rightarrow$ cell structure
    - Higher energy particles: $\lambda = 2\pi/p$
    - X-rays: $\lambda \sim 0.01 - 10 \text{ nm} \rightarrow$ atomic, crystal structure
  - Charged particles:
    - Rutherford experiment: $p \sim 10 \text{ keV}, \lambda \sim 10^{-10} \text{ m}$
    - Discovery of quarks: $p \sim 10 \text{ GeV}, \lambda \sim 0.1 \text{ fm}$
    - LHC: $p \sim 1 - 10 \text{ TeV}, \lambda \sim 10^{-16} - 10^{-17} \text{ m} \rightarrow$ quark
Inclusive Fixed-Target Scattering

Scatter lepton off nucleon (proton or neutron) and measure only the momentum of the scattered lepton.

2 kinematic variables: scattered momentum magnitude \( k' \) and angle \( \theta \)

Define Lorentz-invariant quantities:

\[
Q^2 \equiv -q^2 = -(k - k')^2 = 4EE' \sin^2(\theta/2)
\]

\[
\nu \equiv \frac{p \cdot q}{M} = E - E'
\]

\[
W^2 \equiv (p + q)^2 = M^2 + 2M\nu - Q^2
\]
Elastic Scattering

- No extra particles created, target is not broken apart: $W^2 = M^2$
  - Only one free variable left: $Q^2$
  - Can express the cross section in terms of the cross section for a point-like particle times a *form-factor*, which says something about the structure:

$$
\frac{d\sigma}{d\Omega} = \frac{d\sigma}{d\Omega}\bigg|_{\text{point}} \left| F(Q^2) \right|^2
$$
Meaning of the Form Factor

• For purely Coulomb interaction (spinless target, e.g. a $^{12}$C nucleus) the form-factor is a Fourier transform of charge distribution:

$$F(\vec{q}) = \int d^3r \rho(r) e^{i\vec{q} \cdot \vec{r}}$$

• Angular dependence is trivial. Can expand in terms of small values of $Q^2=-q^2$:

$$F(Q^2) = \int d^3r \rho(r) \left(1 + iq \cdot r - \frac{1}{2} (\vec{q} \cdot \vec{r})^2 + \cdots \right)$$

$$\approx 1 - \frac{1}{6} q^2 \langle r^2 \rangle \quad \text{Charge radius}$$
Nucleon Form Factors

- For nucleons, have to take into account electric (Coulomb) and magnetic (spin-flip) interactions:

\[
\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \theta/2} \frac{E}{E'} \left[ \frac{G_E^2 + \tau G_M^2}{1 + \tau} \left( \cos^2 \theta/2 + 2\tau G_M^2 \sin^2 \theta/2 \right) \right]
\]

\[\tau \equiv \frac{Q^2}{4M^2}\]

Point-like target

Measure form factors by measuring dependence of the cross section on the scattering angle \(\theta\).
Elastic Form Factor

Determine the average charge radius of the proton:

\[
\langle r^2 \rangle = 6 \left( \frac{dG_E(q^2)}{dq^2} \right)_{q^2=0} = (0.81 \text{ fm})^2
\]
Since the lepton and the hadronic system do not interact after scattering, can factorize the cross section into leptonic and hadronic tensors (c.f. calculation for the $e\mu$ scattering cross section)

\[
d\sigma \sim L_{\mu\nu}(k, k') W^{\mu\nu}(p, p')
\]

\[
L_{\mu\nu}(k, k') = 2 \left[ k'_\mu k'_\nu + k'_\nu k'_\mu - (k \cdot k' - m^2) g_{\mu\nu} \right]
\]
Inelastic Scattering (cont.)

Summing over spins, and assuming parity conservation, we can write the most generic form of the hadronic tensor

\[ W^{\mu\nu}(p, q = p' - p) = -W_1 g^{\mu\nu} + \frac{W_2}{M^2} p^\mu p^\nu + \frac{W_4}{M^2} q^\mu q^\nu + \frac{W_5}{M^2} (p^\mu q^\nu + q^\mu p^\nu) \]

Conservation of the EM current leads to two conditions

\[ q_\mu W^{\mu\nu} = q_\nu W^{\mu\nu} = 0 \]

which means only two functions, \( W_1 \) and \( W_2 \), survive. These functions describe the internal structure of the proton and its possible excitations.
Kinematics

Reminder: two independent Lorentz-invariant variables

\[ Q^2 \equiv -q^2 = -(k - k')^2 = 4EE' \sin^2(\theta/2) \]

\[ \nu \equiv \frac{p \cdot q}{M} = E - E' \]

\[ W^2 \equiv (p + q)^2 = M^2 + 2M\nu - Q^2 \]

Inelastic cross section depends on two structure functions:

\[ \frac{d^2 \sigma}{dE'd\Omega} = \frac{\alpha^2}{4E^2 \sin^4 \frac{\theta}{2}} \left[ W_2(\nu, Q^2) \cos^2 \frac{\theta}{2} + 2W_1(\nu, Q^2) \sin^2 \frac{\theta}{2} \right] \]
Stanford Linear Accelerator Center
End Station A: How It All Started

SLAC-R-090 (08/1968)

Experiment E-4: SLAC-MIT-CIT (precursor to discovery of quarks)
First high-power LH2 target at SLAC!
SLAC 8-GeV Spectrometer

P.N. Kirk et al, PRD 8, 63 (1973)
SLAC-PUB-656
Inelastic Scattering Cross Section


**Figure 3.** Spectra of 10 GeV electrons scattered from hydrogen at 6°, as a function of the final hadronic-state energy $W$. Figure (a) shows the spectrum before radiative corrections. The elastic peak has been reduced in scale by a factor of 8.5. The computed radia-
Structure Functions in DIS


\[ x = \frac{1}{\omega} = 0.25 \]

**Figure 17.** $\nu W_2$ for the proton as a function of $q^2$ for $W>2$ GeV, at $\omega=4$.
Results from (7, 8, and 49).
Scaling in DIS

- Consider scattering off point-like target (e.g. muon)

- Scattering is elastic, so

\[ p' = p + q \]

\[ E' = E - \frac{Q^2}{2M} \]

\[ \nu = E - E' = \frac{Q^2}{2M} \]

\[ \nu W_2^{\text{point}} = \delta \left( 1 - \frac{Q^2}{2M} \right) \]

\[ 2MW_1^{\text{point}} = \frac{Q^2}{2M\nu} \delta \left( 1 - \frac{Q^2}{2M} \right) \]
Bjorken Scaling

Define two dimensionless variables:

\[ x = \frac{Q^2}{2p \cdot q} = \frac{Q^2}{2M(E - E')} \]

Bjorken \( x \) (scaling var)

\[ y = \frac{p \cdot q}{p \cdot k} = \frac{E - E'}{E} = 1 - \frac{E'}{E} \]

Fraction of incident energy carried away by hadronic remnants

For point-like targets,

\[ \nu W_2^{\text{point}} = \delta (1 - x) \]

\[ 2M W_1^{\text{point}} = x \delta (1 - x) \]

☞ If the structure functions \( W_{1,2} \) do not depend on \( Q^2 \), but only on Bjorken \( x \), it hints at scattering off point-like objects
Scaling In DIS


\[ x = \frac{1}{\omega} = 0.25 \]

**Figure 17.** $\nu W_2$ for the proton as a function of $q^2$ for $W > 2$ GeV, at $\omega = 4$. Results from (7, 8, and 49).
Nobel Prize in Physics, 1990

Jerome I. Friedman  Henry W. Kendall  Richard E. Taylor

The Nobel Prize in Physics 1990 was awarded jointly to Jerome I. Friedman, Henry W. Kendall and Richard E. Taylor "for their pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics".
Parton Model of DIS

\[ d\sigma \equiv \sum_i \int d\xi_i q_2^2 \]

DIS cross section is an incoherent sum of elastic scattering cross sections off individual constituents (partons). We now these partons as asymptotically free quarks. Parton model should work perfectly for \( Q^2 \to \infty \) where the strong coupling constant is zero. In this limit

\[ MW_1 \to F_1(x) \]
\[ \nu W_2 \to F_2(x) \]
Parton Distribution Functions

• Define parton momentum distribution

\[ f_i(x) \equiv \frac{dP_i}{dx} \quad \text{where} \quad \sum_i \int_0^1 dx f_i(x) = 1 \]

Probability to find a parton with momentum fraction \( x \) of the proton

Then

\[ F_2(x) = \sum_i q_i^2 x f_i(x) \]

\[ F_1(x) = \frac{1}{2x} F_2(x) \quad \text{Callan-Gross relation} \]
Proton Structure Functions

Structure functions are a charge-weighted sum of the quark momentum distributions. Have to account for “valence” quarks (which are always in the proton) and “sea” quarks (which pop out of the gluon field)
Proton and Neutron Structure Functions

\[
\frac{F_2^p(x)}{x} = \left( \frac{2}{3} \right)^2 [u(x) + \bar{u}(x)] + \left( \frac{1}{3} \right)^2 [d(x) + \bar{d}(x)] + \left( \frac{1}{3} \right)^2 [s(x) + \bar{s}(x)]
\]

(忽略 charm 和 bottom 夸克，它们的贡献很小，除了在非常高能，例如 LHC)

Isospin symmetry implies interchange \(u^p(x) \leftrightarrow d^n(x)\):

\[
\frac{F_2^n(x)}{x} = \left( \frac{2}{3} \right)^2 [d(x) + \bar{d}(x)] + \left( \frac{1}{3} \right)^2 [u(x) + \bar{u}(x)] + \left( \frac{1}{3} \right)^2 [s(x) + \bar{s}(x)]
\]
Valence Quark Distributions

\[ u_V(x) \equiv u(x) - \bar{u}(x) \]
\[ d_V(x) \equiv d(x) - \bar{d}(x) \]

Expect maximum to be near \( x \sim 1/3 \)

Sum rules:

\[ \int_0^1 dx u_V(x) \equiv \int_0^1 dx u(x) - \bar{u}(x) = 2 \]
\[ \int_0^1 dx d_V(x) \equiv \int_0^1 dx d(x) - \bar{d}(x) = 1 \]
\[ \int_0^1 dx s_V(x) \equiv \int_0^1 dx s(x) - \bar{s}(x) = 0 \]
Valence Quark Distribution

J.I. Friedman and H.W. Kendall, 

\[ \nu(W_2^p - W_2^n) = F_2^p - F_2^n = \frac{x}{3} [u_V(x) - d_V(x)] \]

**Figure 24.** Preliminary values of \((W_2^p - W_2^n)\) (smeared) as a function of \(x = 1/\omega\) derived from the points of Figure 22 with the assumption that \(R_n = R_p\). This data is discussed further in the text.
Proton vs Neutron

\[ F_2^n/F_2^p \]

Sea dominates

\[ u_V \] dominates

J.I. Friedman and H.W. Kendall,

**Figure 22.** The quantity \( D/H_s - 1 \) as a function of \( x = 1/\omega \). The data are from (19), and are for \( W > 2 \) GeV and \( q^2 > 1 \) (GeV/c)^2. This quantity is approximately equal to the ratio of neutron to proton scattering cross sections. The ratios shown are averages over small intervals of \( x \). See also Figure 23.
Sea Quarks

Sea quarks are produced by splitting of the gluons into quark-antiquark pairs. Gluons are in turn radiated off valence quarks… Both of these processes make sea quarks very soft (small momentum, which means small $x$).

Expect probability to find a sea quark to increase steeply at low $x$. 
Photon-Nucleon Cross Section

\[ \sigma_{tot}(\gamma p) \sim \nu^{0.08} \]
Unpolarized Structure Functions

From PDG08

Q^2 = 12 GeV^2
Q^2 = 4.5 GeV^2
Q^2 = 2.5 GeV^2
Q^2 = 1.4 GeV^2
Q^2 = 0.65 GeV^2
Q^2 = 0.045 GeV^2

Proton
- ZEUS
- H1
- SLAC
- BCDMS
- NMC
- ZEUS Regge
- H1 QCD

Scaling violations!

sea quarks dominate

valence quarks dominate
Figure 16.4: Distributions of $x$ times the unpolarized parton distributions $f(x)$ (where $f = u_v, d_v, \bar{u}, \bar{d}, s, c, b, g$) and their associated uncertainties using the NNLO MRST2006 parameterization [13] at a scale $\mu^2 = 20 \text{ GeV}^2$ and $\mu^2 = 10,000 \text{ GeV}^2$. 
Perturbative QCD

- QCD is based on the SU(3) symmetry of color
  - Non-abelian group: gauge carriers charged under group
  - 3 quark colors, 8 colored gluons
  - Predicts asymptotic freedom
    - Gross, Wilczek, Politzer (1973)

- Perturbative QCD: perturbative expansion in terms of $\alpha_s$
  - Log dependence on $Q^2$: scaling violations in DIS
  - Confirmed by:
    - Measuring running of $\alpha_s(Q^2)$
    - QCD sum rules
    - $Q^2$ evolution of parton distributions
    - $e^+e^-$ annihilation into hadrons, jets, etc.
Nobel Prize in Physics, 2004

David J. Gross       H. David Politzer       Frank Wilczek

The Nobel Prize in Physics 2004 was awarded jointly to David J. Gross, H. David Politzer and Frank Wilczek "for the discovery of asymptotic freedom in the theory of the strong interaction".

Photos: Copyright © The Nobel Foundation
Momentum Sum Rule

\[ \int_0^1 dx \ x [u + \bar{u} + d + \bar{d} + s + \bar{s}] \equiv \varepsilon_q \]

Momentum fraction carried by all quarks

Basic momentum sum rule: \( \varepsilon_q + \varepsilon_g = 1 \)

Naively, would expect \( \varepsilon_q \approx 1 \). However, DIS measures

\[ \begin{align*}
\int_0^1 F_2^p(x) dx &= \frac{4}{9} \varepsilon_u + \frac{1}{9} \varepsilon_d \approx 0.18 \\
\int_0^1 F_2^n(x) dx &= \frac{4}{9} \varepsilon_d + \frac{1}{9} \varepsilon_u \approx 0.12
\end{align*} \]

\( \varepsilon_q = 0.54 \) at low \( Q^2 \) (ignoring strange quarks) with error of about 10%.

\( \varepsilon_q = 0.48 \) predicted in 1974 by Gross and Wilczek from perturbative QCD (PRD 9, 980 (1974)): consequence of the color structure of SU(3). First indirect evidence for the gluon
Gottfried Sum Rule

\[ \int_0^1 \frac{F_2^p(x) - F_2^n(x)}{x} \, dx = \frac{1}{3} + \frac{2}{3} \int_0^1 [\bar{u}(x) - \bar{d}(x)] \, dx = \frac{1}{3} \]

Valid for symmetric sea, i.e. \( \bar{u}(x) = \bar{d}(x) \)

Experimentally:

\[ \int_{0.004}^{0.8} \frac{F_2^p(x) - F_2^n(x)}{x} \, dx = 0.2281 \pm 0.0065 \quad \text{(NMC, } Q^2=4 \text{ GeV}^2) \]

Interpretation: sea quark distributions are asymmetric, i.e.

\( \bar{u}(x) \neq \bar{d}(x) \neq \bar{s}(x) \)

Most likely consequence of small quark mass difference
Scaling Violations

- At finite $Q^2$, the strong coupling constant $\alpha_S$ is not small
  - Corrections of order $O(\alpha_S)$ can be important
  - For example, here are some contributions to DIS

\[
\begin{array}{c}
\text{real} \quad \frac{x}{y} \quad k_t \\
\text{virtual}
\end{array}
\]

- The virtual diagrams typically lead to $O(1+\alpha_S(Q^2)/\pi)$ factor in the cross section (log dependence of $F_2$ on $Q^2$)
- First term is qualitatively different. It changes the momentum fraction $x$ of the parton. In other words, it evolves parton PDF $f_i(x)$
- To find the distribution sampled by the photon, need to integrate over all $y$, or equivalently, all gluon momenta
Taking into account gluon radiation, we can write

\[
\frac{F_2(x, Q^2)}{x} = \sum_i q_i^2 [f_i(x) + \delta f_i(x, Q^2)]
\]

where

\[
\delta f_i(x, Q^2) = \frac{\alpha_s}{2\pi} \log \frac{Q^2}{\mu^2} \int_x^1 \frac{dy}{y} f_i(y) P_{qq}(x/y)
\]

Can rewrite it as a differential equation:

\[
\frac{df_i(x, Q^2)}{d \log Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dy}{y} f_i(y) P_{qq}(x/y)
\]  

PDF measured at \(Q^2=\mu^2\)

"splitting function"

some arbitrary momentum scale

quark-quark
DGLAP Equations

Yuri Dokshitzer  
Vladimir Gribov  
Lev Lipatov  
Guido Altarelli  
Giorgio Parisi

Sov.Phys. JETP 46, 641 (1977) 
ЯЭТФ 73, 1216 (1977)

Describe evolution of the quark and gluon distribution functions with $Q^2$

ASYMPTOTIC FREEDOM IN PARTON LANGUAGE

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Received 12 April 1977

A novel derivation of the $Q^2$ dependence of quark and gluon densities (of given helicity) as predicted by quantum chromodynamics is presented. The main body of predictions of the theory for deep-inelastic scattering on either unpolarized or polarized targets is re-obtained by a method which only makes use of the simplest tree diagrams and is entirely phrased in parton language with no reference to the conventional operator formalism.
**DGLAP Equations**

\[
\frac{\partial q_v(x, Q^2)}{\partial \log Q^2} = \frac{\alpha_s}{2\pi} P_{qq} \otimes q_v
\]

\[
\frac{\partial}{\partial \log Q^2} \left( \begin{array}{c} \Sigma \\ g \end{array} \right) = \frac{\alpha_s}{2\pi} \left( \begin{array}{cc} P_{qq} & 2n_f P_{qg} \\ P_{gq} & P_{gg} \end{array} \right) \otimes \left( \begin{array}{c} \Sigma \\ g \end{array} \right)
\]

\[
\Sigma \equiv \sum_q q + \bar{q}
\]

where

\[
q_v \equiv q - \bar{q}
\]

and

\[
P \otimes q \equiv \int_x^1 \frac{dy}{y} P(x/y)q(y)
\]

Interpretation: quark and gluon PDFs feed off each other, increasing at larger \(Q^2\) and smaller \(x\). This introduces logarithmic scaling violations to DIS. DIS is sensitive to gluon distributions at \(O(\alpha_s)\).

Can factorize the cross section:

\[
\begin{array}{c}
\text{Non-perturbative PDFs cannot be computed} \\
\text{“Hard-scattering” cross sections: can be computed perturbatively}
\end{array}
\]
Figure 16.4: Distributions of $x$ times the unpolarized parton distributions $f(x)$ (where $f = u_v, d_v, \bar{u}, \bar{d}, s, c, b, g$) and their associated uncertainties using the NNLO MRST2006 parameterization [13] at a scale $\mu^2 = 20 \text{ GeV}^2$ and $\mu^2 = 10,000 \text{ GeV}^2$. 
Spin (Polarized) Structure Functions

Scattering of polarized beam off polarized targets

\[ A_{LL} \equiv \frac{\sigma^{\uparrow \downarrow} - \sigma^{\uparrow \uparrow}}{\sigma^{\uparrow \downarrow} + \sigma^{\uparrow \uparrow}} \simeq \frac{g_1(x, Q^2)}{F_1(x, Q^2)} \]

\[ g_1(x, Q^2) = \frac{1}{2} \sum_q e_q^2 (\Delta q + \Delta \bar{q}) \]

\[ \Delta q \equiv q^{\uparrow} - q^{\downarrow} \]

Spin sum rule:

\[ \frac{1}{2} \equiv \frac{1}{2} \Delta \Sigma + \Delta G + L_z^q + L_z^G \]

Experiment: \( \Delta \Sigma \approx 0.3 \)

(this used to be known as “Spin Crisis” of parton model)

The rest (70%) of the spin has to come from gluons and orbital angular momentum

YGK, Hadronic Structure
Figure 16.5: Distributions of $x \Delta q(x)$ (where $q = u, d, \bar{u}, \bar{d}, s$) in the LSS 2006 [45], AAC 2008 [46], and DSS 2008 parameterizations at a scale $\mu^2 = 2.5 \text{GeV}^2$, showing error correction of the latter set (corresponding to a one-unit increase in $\chi^2$). Points represent data from semi-inclusive positron (HERMES [51,52]) and muon (SMC [53] and COMPASS [54]) deep inelastic scattering given at $Q^2 = 2.5 \text{GeV}^2$. SMC results are extracted under the assumption that $\Delta u(x) = \Delta d(x)$.

The hadronic photon structure function, $F_{\gamma^2}$, evolves with increasing $Q^2$ from the 'hadron-like' behavior, calculable via the vector-meson-dominance model, to the dominating 'point-like' behaviour, calculable in perturbative QCD. Due to the point-like coupling, the logarithmic evolution of $F_{\gamma^2}$ with $Q^2$ has a positive slope for all values of $x$, see Fig. 16.14. The 'loss' of quarks at large $x$ due to gluon radiation is over-compensated by the 'creation' of quarks via the point-like $\gamma \to q \bar{q}$ coupling. The logarithmic evolution was first predicted in the quark–parton model ($\gamma^* \gamma \to q \bar{q}$) [63, 64], and then in QCD in the limit of large $Q^2$ [65]. The evolution is now known to NLO [66–68]. NLO data...
Gluon contribution to the spin

Understanding the gluon is crucial for the proton structure

Extracted via semi-inclusive processes: meson production in polarised DIS and pp (RHIC)

Global pol-analysis: extract polarised PDF's

Extreme options now excluded
Extend x-range in pp at RHIC
e^+e^- Annihilation into Hadrons

Recall from QED

\[ \sigma(e^+e^- \rightarrow \mu^+\mu^-) = \frac{4\pi\alpha^2}{3s} \approx \frac{87 \text{ nb}}{s \text{ (GeV)}^2} \]

What is the difference for quarks? Charge, and the fact that they are confined (not free). If they were free, cross section would be:

\[ \sigma(e^+e^- \rightarrow q\bar{q}) = \frac{4\pi\alpha^2}{3s} \cdot q^2 \cdot 3 \quad \text{count 3 colors} \]

How do we account for hadronization, i.e. production of hadrons from quarks?

**Quark-hadron duality theorem**: the sum over *all possible* hadronic final states away from bound states is equivalent to quark cross section. Basically, it means that probability for a quark to hadronize into *something* is unity.
**e^+e^- Annihilation into Hadrons**

\[ \sigma(e^+e^- \rightarrow q\bar{q}) = \frac{4\pi\alpha^2}{3s} \sum_i 3q_i^2 \]

Taking into account corrections of \(O(\alpha_s)\) due to gluon radiation,

\[ R \equiv \frac{\sigma(e^+e^- \rightarrow q\bar{q})}{\sigma(e^+e^- \rightarrow \mu^+\mu^-)} = 3 \sum_i q_i^2 \left(1 + \frac{\alpha_s}{\pi}\right) \]

Here the sum is over all quarks that can be produced at the CM energy of the collision, i.e. \((2m_q)^2 < s\)

Ratio \(R\) measures the number of colors, and quark masses, i.e. fundamental parameters of QCD
Hadronic R Ratio

\[
\begin{align*}
(u,d,s): R &\approx 2 \\
(u,d,s,c): R &\approx \frac{10}{3} \\
(u,d,s,c,b): R &\approx \frac{11}{3}
\end{align*}
\]