FINAL EXAMINATION
Friday, December 14, 5–8pm

Maximum score: 200 points

You are given 180 minutes for this exam. You are allowed three $8\frac{1}{2}'' \times 11''$ sheets containing any information you wish on both sides. Derive all answers symbolically, then plug in the numbers, if appropriate. You may leave expressions such as $\sqrt{2}$, $e$, $\pi$ unevaluated. You may use a calculator, but if you do not have one, estimate an order of magnitude and write down dimensions for your answer. Your description of the physics involved and symbolic answers are worth much more than the numeric answers. Show all work, and take particular care to explain what you are doing. Write directly on the exam, and if you need extra pages, make sure to put a note on the corresponding sheet. Cross out rather than erase parts of the problem you wish the grader to ignore. Box or circle final answers.

There are six problems with points assigned as shown. Partial credit will be given for incomplete solutions, so attempt to do all problems. Some problems will take significantly longer than others, so judge time appropriately. At the beginning of the exam, please look through all problems and plan how you’ll spend your time.

If you need to ask a question, come to the proctor. Your question and the answer will be written on the board.

Read all problems carefully.
If you get stuck on a problem, move to another.
Try to remain calm and work steadily. You’ve almost made it!

Good luck!

NAME: ____________________________________________

SID #: ____________________________________________
1. (20 points) A cylinder of radius $R$ is placed between two non-slippery surfaces (see picture) which are moving with velocities $v_1$ and $v_2$. Find the velocity of the center of the cylinder $V$ and its angular velocity $\omega$ if

(a) The surfaces move in the same direction

(b) The surfaces move in the opposite directions
2. (20 points) A wooden block of mass $M$, initially at rest on a table with coefficient of sliding friction $\mu$, is struck by a bullet of mass $m$ and velocity $v$. The bullet lodges in the center of the block. How far does the block slide?
3. *(40 points)* A man begins to climb up a 12-ft ladder (see figure). The man weighs 180 lb, and the ladder 20 lb. The wall against which the ladder rests is very smooth, which means that the tangential (vertical) component of force at the contact between ladder and wall is negligible. The foot of the ladder is placed 6 ft from the wall. The ladder, with the man’s weight on it, will slip if the tangential (horizontal) force at the contact between the ladder and ground exceeds 80 lb. How far up the ladder can the man safely climb?
4. (30 points) A neutral particle $\pi^0$ meson decays symmetrically into two photons while moving at high speed. The energy of each photon in the laboratory frame is 100 MeV. The mass of the $\pi^0$ is $m = 135$ MeV.

(a) Find the velocity of the meson $V$

(b) Find the angle between the photons in the LAB frame
5. (50 points) Imagine a straight tunnel bored through the center of the Earth from one point on the surface (say, near Berkeley, CA) to the other side (alas, this would be in the Indian Ocean somewhere!). After the tunnel is evacuated and the inside surface is coated so that there is little resistance, an H7A student releases a stone into the hole without any initial velocity. Assume that the Earth is a uniformly dense sphere.

(a) Show that in the absence of friction, the motion of the rock is periodic, and find the period.

(b) Suppose that instead of going through the middle of the Earth, the tunnel were bored distance \( h = 1000 \text{ km} \) from the center of the Earth. Also suppose that the wall coating is not perfect, and introduces friction with a coefficient dry friction \( \mu = 0.1 \). If we threw the rock into such tunnel, how far from the surface of the Earth on the other side would it reach?
6. (40 points) Two longitudinal waves are propagating in the elastic medium: one along $X$ direction, and another along $Y$ direction. The waves described by the equations

$$
\xi_x = a \cos \frac{2\pi}{\lambda} (x - vt)
$$

$$
\xi_y = a \cos \frac{2\pi}{\lambda} (y - vt)
$$

where $\xi_x$ and $\xi_y$ are displacements along $X$ and $Y$ axes, respectively. Find in what regions of space the trajectories of particles of the medium would be

(a) Linear

(b) Circular