1 Disc in a Wheel

Consider a wheel of radius $R$ which can be rotated freely along the axis perpendicular to the plane of the wheel. Inside the wheel, we place a disc of mass $m$ and radius $r$. Assume that the static friction between the surfaces of the wheel and the disc is large enough such that the disc does not slip at any time.

a. Suppose that we rotate the wheel at a constant angular velocity $\Omega_0$, what is the position and the angular velocity of the disc? What is the total kinetic energy of the system? (What is the total angular momentum of the system?)

b. Suppose that we rotate the wheel at a constant angular acceleration $\alpha$, what will be the equilibrium position of the disc (angle $\theta$ measured from the vertical line) when the wheel is rotated for time long enough? Is it possible to have $\theta = 90^\circ$?

2 Force of Water

You want to hold a slab made of wood against the wall by shooting water at it. The coefficient of static friction between the slab and the wall is given by $\mu_s$. If the mass of the slab is $m$, the area of the water hose is $A$ and the density of the water is $\rho$, what is the smallest speed of the water required to hold the slab? Assume that after the water hits the slab, the water then move completely upwards or downwards.
3 Static and Kinetic Friction

We all know that the coefficient of static friction, \( \mu_s \), is greater than that of kinetic friction, \( \mu_k \). What is wrong with having \( \mu_s < \mu_k \)? Think of the situation where \( \mu_s < \mu_k \) as you increase the force applied to an object on a table.

4 Hanging “L”

Consider a metal bar shaped like an “L” as shown in Figure 3. The bar is suspended from a ceiling by a string. Find the angle \( \theta \) that the bar makes with the string when it is in equilibrium using

a. center-of-mass approach.

b. torque equation approach.

5 Sliding on the Floor

Suppose you gradually lower a rotating disc of mass \( m \) and radius \( r \) on a surface with coefficient of kinetic friction of \( \mu_k \) and static friction of \( \mu_s \), and let go. If the initial angular speed of the disc is \( \omega_0 \), of course initially the disc will slip on the floor. However, eventually the disc will reach a final linear speed (and some angular speed) such that it just rolls without slipping. Find

a. the final speed of the center-of-mass of the disc. How does this speed depend on \( \mu_k \)?

b. the distance traveled by the disc from the position where you lower the disc to the point where it reaches the final speed calculated in part a.

6 New Moon

If somehow the mass of the moon is increased to be the same as the mass of the earth but the moon-earth distance does not change, how many days are there in a lunar
month? Use the following parameters:

- Mass of the earth = $5.9 \times 10^{24}$ kg.
- Mass of the moon = $7.4 \times 10^{22}$ kg.
- Earth-moon distance (center-to-center) = $3.8 \times 10^{8}$ m.
- $G = 6.67 \times 10^{-11}$ m$^3$ kg$^{-1}$s$^{-2}$.

7 Air Resistance

Assume that the force from air resistance acting on the moving object can be written as $f_{\text{air}} = -k|\vec{v}|\hat{v}$ where $k$ is a constant and $\vec{v}$ is the velocity of the object. Now you throw an object directly upward with initial speed $v_0$. Compare the times it takes to go up (from your hand to the highest point in the object’s trajectory) and come down (from the highest point back to you hand). Which time is longer? or are they the same?