Precision measurements and ‘New Physics’

William J Marciano

Brookhaven National Laboratory, Upton, NY 11973, USA
and
Institute für Theoretische Teilchenphysik, Universität Karlsruhe, D-76128 Karlsruhe, Germany

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Abstract
The role of precision measurements in testing the standard model and probing for ‘New Physics’ is described. Anomalous magnetic moments of the electron ($a_e$) and muon ($a_\mu$) along with the radiative correction functions $\Delta r$ and $\Delta \hat{r}$ that relate $\alpha$, $G_F$, $m_Z$, $m_W$ and $\sin^2 \theta_W(m_Z)$ are discussed. Current discrepancy between $a_\mu$ theory and experiment is correlated with constraints on the Higgs mass from $m_W$ and $\sin^2 \theta_W(m_Z)$ obtained from Z pole lepton asymmetries. Together, they suggest hints of ‘New Physics’ may be starting to appear in quantum loop effects.

1. Introduction

Whenever I give a general lecture on precision measurements, I like to begin with the following quote:

"The more important fundamental laws and facts of Physical Science have all been discovered and these are now so firmly established that the possibility of their ever being supplanted in consequence of new discoveries is exceedingly remote.... Our future discoveries must be looked for in the sixth place of decimals." A Michelson (1894)

That famous statement by Michelson is often misinterpreted as an end of physics lament. It is then pointed out that just two years later in 1896 Becquerel’s discovery of radioactivity ushered in the age of modern physics. Furthermore, in 1897 the electron was discovered, giving birth to elementary particle physics and eventually our great technological revolutions. Those discoveries were soon accompanied by an intellectual Renaissance in theory that included special relativity (1905), general relativity (1916) and quantum mechanics (1925). It would appear that Michelson’s foresight was far off the mark.

J J Sakurai Prize Lecture, APS Meeting, Albuquerque, NM April 2002. Contents of the original talk have been updated to include recent results.
However, Michelson’s message was meant to extol the virtues of precision measurements rather than to despair the state of physics. Interestingly, it is more appropriate today than it was 108 years ago.

Michelson, of course, had made his fame by measuring the speed of light with ever more precision. His pioneering work helped inspire special relativity and like most precision measurement efforts, advanced technological innovation and creativity. Michelson died while trying to push his measurement of the speed of light to the sixth place. Today, we know the speed of light to 1 part/billion!

In fact,

\[ c = 299,792,458 \text{ m s}^{-1} \]  

(1)

exactly [1], since that relation is now used to define the meter. What would Michelson have thought of that?

Of course, during the century that followed Michelson, precision measurements took several new directions. Quantum electrodynamics (QED) and then electroweak unification allowed high order quantum loop calculations of physical quantities. One could test theory and search for deviations due to ‘New Physics’ by ever more precise experimental measurements. In the case of pure QED, experiments have been pushed to 1 part/billion precision, without seeing a breakdown in theory, a marvelous feat. In the case of electroweak studies, the parameter space of related observables is much richer and although experiments have ‘only’ probed the ±0.1% level, they are in fact more sensitive to ‘New Physics’ effects because their starting point, the scale of weak interactions is at much shorter distances.

In this paper, I will describe several types of precision studies that I have been involved in, contrasting their probing potential while at the same time pointing out connections between them. I will begin with the anomalous magnetic moments of the electron (\(a_e\)) and the muon (\(a_\mu\)). In both cases, heroic experimental efforts [2, 3] have been carried out. For the electron, \(a_e\) is used to provide our best determination [4] of the fine structure constant, \(\alpha\). For the muon, a 3 sigma difference between experiment and theory currently exists [5]. It could be a harbinger of ‘New Physics’ loop effects [6] or a computational problem involving e\(^+\)e\(^-\) data input. (Of course, the \(a_\mu\) experiment might also be at fault). I then shift gears and discuss electroweak precision tests which compare \(\alpha, G_\mu\) (the Fermi constant), \(m_z, m_w\), and \(\sin^2 \theta_w(m_z)\) via natural standard model relationships among those otherwise distinct parameters. Such tests are embodied in the radiative corrections functions [7] \(\Delta r\) and \(\Delta \hat{r}\) which depend [8] on the as yet unknown Higgs scalar mass as well as potential ‘New Physics’ at very high mass scales [9]. Those quantities are currently used to constrain the Higgs mass, and suggest that it may be relatively light, i.e. very close to direct experimental bounds. In fact, some of the best precision measurements, \(m_w\) and \(\sin^2 \theta_w(m_z)\) (obtained from leptonic Z pole asymmetry measurements) seem to predict too low a Higgs mass, thus suggesting that the presence of ‘New Physics’ may be causing a small distortion. Finally, I will try to connect the deviation in \(a_\mu\) with those in \(m_w\) and \(\sin^2 \theta_w(m_z)\). The connection is hadronic vacuum polarization. To bring theory and experiment together requires a larger hadronic vacuum polarization contribution, a possibility suggested by \(a_\mu\) tau decay data. However, a larger hadronic vacuum polarization would increase the discrepancy between the \(m_w\) and \(\sin^2 \theta_w(m_z)\) prediction for the Higgs mass, \(m_H\), and the experimental bound on it. So, it seems that, collectively, these very different measurements may be constraining one another and hinting at the presence of ‘New Physics.’

2. Electron anomalous magnetic moment

QED is a renormalizable quantum field theory. Precise calculations can be carried out perturbatively using the fine structure constant \(\alpha = e^2 / 4\pi \simeq 1/137\) as an expansion parameter.
Predictions based on that prescription have been tested to very high accuracy in the now classic QED effects such as the Lamb shift, hyperfine splitting etc \[10\]. One of the best tests is provided by the electron anomalous magnetic moment \(a_e\). Higher orders have been computed over the years such that the prediction now stands at \[4, 11\]

\[
a_e = \frac{\alpha}{2\pi} + 0.328\ 478\ 444\ (\frac{\alpha}{\pi})^2 + 1.181\ 234\ (\frac{\alpha}{\pi})^3 - 1.7502\ (\frac{\alpha}{\pi})^4 + \cdots + 1.66\times 10^{-12} \tag{2}
\]

where the last term represents hadronic and weak loop effects \[11\] which are very small. I should note that the coefficient of the \((\alpha/\pi)^4\) term has recently undergone a small revision which is included in equation (2) \[4\].

On the experimental side, Dehmelt and his collaborators have found in a series of beautiful experiments \[2\]

\[
\begin{align*}
a_{e\text{exp}} &= 0.001\ 159\ 652\ 1884(43) \\
a_{e\text{exp}} &= 0.001\ 159\ 652\ 1879(43) \tag{3}
\end{align*}
\]

where the bracketed number represents the uncertainty in the last two digits.

The precision is extraordinary. However, to compare equations (2) and (3) requires an independent determination of \(\alpha\) with similar precision. Currently, one finds from the best condensed matter and atomic measurements \[12\]

\[
\begin{align*}
\alpha^{-1} &= 137.036\ 003\ 00(270) \quad \text{Quantum Hall} \\
\alpha^{-1} &= 137.036\ 008\ 40(330) \quad \text{Rydberg (h/m_e)} \\
\alpha^{-1} &= 137.035\ 987\ 10(430) \quad \text{AC Josephson} \tag{4} \\
\alpha^{-1} &= 137.035\ 995\ 20(790) \quad \text{Muonium HFS} \\
\alpha_{\text{average}} &= 137.036\ 001\ 40(183).
\end{align*}
\]

Those values are not precise enough (by about an order of magnitude) to fully exploit the theory and experimental efforts in equations (2) and (3). Instead, one generally derives a value for \(\alpha\) \[4\] by equating equations (2) and (3)

\[
\alpha^{-1} = 137.035\ 998\ 77(40) \tag{5}
\]

which is the best determination of that fundamental parameter. It is in relatively good agreement with the values in equation (4) (differing by 1.4 sigma from \(\alpha^{-1}\) average), thereby testing the validity of QED with extraordinary precision. A new experiment \[13\] aims to improve \(a_{e\text{exp}}\) by an order of magnitude. To fully utilize such an advance in the search for ‘New Physics’ or a breakdown of QED will require a much better independent measurement of \(\alpha\).

Although \(a_e\) represents an incredibly precise test of QEDs validity, it is not particularly sensitive to ‘New Physics.’ Generally, one expects such effects in \(a_e\) to be of orders of \((m_e^2/\Lambda^2)\) where \(\Lambda\) is the scale of ‘New Physics.’ (There may be, of course, further \(\alpha/\pi\) suppression factors.) Indeed the small hadronic and weak effects in equation (2) correspond to mass scales \(m_\rho \simeq 770\ \text{MeV} \) and \(m_w \simeq 80\ 4\ \text{GeV}\), along with coupling factor suppressions. So, at present, one finds \(a_e\) probes for ‘New Physics’ at the \(\Lambda \lesssim 10\ \text{GeV}\) level \[6, 11\], not a very high scale by today’s standards.

3. Muon anomalous magnetic moment

The muon is about 207 times heavier than the electron. Therefore, all else being roughly equal, one expects ‘New Physics’ effects in \(a_\mu\) to be about \(m_\mu^2/m_e^2 \sim 40\ 000\) times larger than in \(a_e\). Since \(a_{\mu\text{exp}}\) is only about 100 times less precise than \(a_{e\text{exp}}\), that means it probes scales about 20 times higher, reaching into the interesting TeV regime.
Recently, experiment E821 at Brookhaven reported [3] an improved determination of
\[ a_{\mu}^{\exp} = 116.592.030(80) \times 10^{-11}. \]  
(6)

Additional \( \mu^- \) data still under analysis could further lower the uncertainty to about \( \pm(60–65) \times 10^{-11} \). Additional (highly warranted) running of the experiment would allow the error to be reduced to about \( \pm40 \times 10^{-11} \), the original proposed goal of the experiment.

Already, the finding of equation (6) has potentially very interesting implications for ‘New Physics.’ However, to fully utilize it requires an equally precise standard model prediction for \( a_{\mu} \). Now, the hadronic and weak loop effects are also about \( m_{\mu}^2/m_e^2 \simeq 40,000 \) times larger than in \( a_e \) and must be very accurately determined. Here, I only present the current theoretical predictions with brief commentary.

The standard model prediction for \( a_{\mu} \) is given by
\[ a_{\mu}^{\text{SM}} = a_{\mu}^{\text{QED}} + a_{\mu}^{\text{EW}} + a_{\mu}^{\text{Hadronic}}. \]  
(7)

QED loop effects have been computed through orders of \( (\alpha/\pi)^4 \) and the leading 5 loop corrections have been estimated. The prediction currently stands at (using \( \alpha^{-1} \) from \( a_e \) as input) [11]
\[ a_{\mu}^{\text{QED}} = 116.584.706(3) \times 10^{-11}. \]  
(8)

I note that the coefficient of the \( \theta(\alpha/\pi)^4 \) term in \( a_{\mu}^{\text{QED}} \) is being re-examined [4] and some shift is expected. The change will move the value somewhat, but it should not significantly impact comparison of theory and experiment.

In the case of electroweak effects, complete one- and two-loop calculations have been carried out. The two-loop contribution turned out to be unexpectedly large (about \(-22\% \) of the loop) [14]. For that reason, it became important to examine leading log 3 one loop electroweak effects. They turned out to be negligibly small [15]. Currently, the electroweak contribution is confidently given by
\[ a_{\mu}^{\text{EW}} = 152(1)(2) \times 10^{-11}. \]  
(9)

Hadronic contributions first enter at the two-loop level, orders of \( (\alpha^2) \). They cannot be evaluated by first principles QCD calculations, at least not yet. Instead, one uses precisely measured cross sections for \( (\sigma(e^+e^- \to \text{hadrons}) \) in a dispersion relation to evaluate hadronic vacuum polarization. That procedure requires non-trivial QED corrections to the \( e^+e^- \) data as well as the reaction used to normalize the machine luminosity. A recent updated analysis by Davier et al [5] found
\[ a_{\mu}^{\text{Had}}(\text{vac.pol.}) = 6847(70) \times 10^{-11} \quad (e^+e^- \text{ data}). \]  
(10)

Improved precision was made possible by new data from Novosibirsk for \( e^+e^- \to \pi^+\pi^- \) in the important \( \sqrt{s} \simeq 0.61–0.96 \) GeV region where \( \pm6\% \) accuracy was achieved [16].

The value in equation (10) is not without controversy. It is lower by \(-77 \times 10^{-11} \) than the favoured 1998 value [17] in which both \( e^+e^- \to \text{hadrons} \) and \( \tau \to \nu_{\tau} + \text{hadrons} \) (along with isospin violation corrections) were used in the dispersion relation. In fact, the tau decay data is now inconsistent with \( e^+e^- \) data even after all ‘known’ isospin violation corrections are applied. The difference is illustrated by the counterpart of equation (10) found by replacing \( e^+e^- \) data by tau data (for \( \sqrt{s} \lesssim 1.77 \) GeV).
\[ a_{\mu}^{\text{Had}}(\text{vac.pol.}) = 7046(62) \times 10^{-11} \quad (\tau \text{ data}) \]  
(11)

where I have taken the liberty to increase the value given by Davier et al [5] by \(+27 \times 10^{-11} \) (a +0.5% shift in the tau derived part). The difference stems from my use of smaller electroweak
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radiative corrections [18] than those employed in [5] and \( V_{ud} = 0.9740 \) rather than 0.9752 employed in [5].

The difference between equations (10) and (11) represents a serious impediment to the comparison of \( a^\text{exp}_\mu \) with \( a^\text{SM}_\mu \) and must be resolved. Furthermore, as I later pointed out, the differences between \( e^+e^- \to \text{hadrons} \) and \( \tau \to \nu_\tau + \text{hadrons} \) data also complicates the Higgs mass constraints from other precision measurements such as \( m_w \) and \( \sin^2 \theta_W(m_Z) \), an additional reason to reconcile the problem.

Hadronic contributions to \( a_\mu \) at orders of \( (\alpha^3) \) are smaller, but have a jaded history. In particular, the so-called light by light hadronic contribution has undergone about 3 sign changes in the 25 year history of its evaluation. The sign now appears to be correct [19] and one finds in total a relatively small three-loop effect.

\[
a^\text{Had}_{\mu}(3\text{loop}) = -14(35) \times 10^{-11}. \tag{12}
\]

That value represents a major shift of \( +171 \times 10^{-11} \) compared to last year. Sign mistakes are usually trivial in origin, but they cause 200\% changes! Together equations (10) and (12) give

\[
a^\mu_\text{Had} = 6833(70) \times 10^{-11} \tag{13}
\]

but remember tau data suggests a larger \( a^\text{Had}_\mu \) by \( +199 \times 10^{-11} \), a major difference.

Adding the contributions in equations (8), (10) and (13), one finds [4]

\[
a^\mu_\text{SM} = 116,591,693(78) \times 10^{-11} \tag{14}
\]

where the errors have been added in quadrature. Comparing that value with \( a^\text{exp}_\mu \) in equation (6) results in a 3 sigma deviation between experiment and theory

\[
a^\text{exp}_\mu - a^\text{SM}_\mu = 337 \pm 112 \times 10^{-11}. \tag{15}
\]

Such a deviation is ripe for speculation on what ‘New Physics’ may be responsible. I subsequently mention the leading candidate, supersymmetry. However, one should keep in mind that tau data used in place of \( e^+e^- \) data would give

\[
a^\text{exp}_\mu - a^\text{SM}_\mu (\tau \text{ data}) = 138 \pm 107 \times 10^{-11} \tag{16}
\]

a much less compelling 1.3 sigma deviation.

Assuming that equation (15) is correct, what ‘New Physics’ could explain such a large deviation? Indeed, the difference \( 337 \times 10^{-11} \) is more than twice the electroweak contribution in equation (9)! Could ‘New Physics’ show up in \( a_\mu \) before weak interaction effects from the W and Z bosons? The answer is yes. In fact, many viable scenarios have been suggested [6]. However, the leading candidate is supersymmetry.

For illustration purposes, I assume a single mass \( m_{\text{susy}} \), for sleptons, sneutrinos and gauginos that enter the \( a_\mu \) calculation at the 1 (and 2) loop level. Then one finds [20] (including leading 2 loop effects)

\[
a^\text{susy}_\mu \simeq \text{sgn}(\mu) \times 130 \times 10^{-11} \left( \frac{100 \text{ GeV}}{m_{\text{susy}}} \right)^2 \tan \beta \tag{17}
\]

where \( \text{sgn}(\mu) = \pm \) is the sign of the \( \mu \) term in supersymmetry models and \( \tan \beta > 3-4 \) is the ratio of the two scalar vacuum expectation values, \( \tan \beta = \langle \phi_2 \rangle / \langle \phi_1 \rangle \). The \( \tan \beta \) factor is an important source of enhancement. As experimental constraints on the Higgs mass have increased, so has the lower bound on \( \tan \beta \). With larger \( \tan \beta \) now required, it appears inevitable that SUSY loops have a fairly major effect on \( a_\mu \) if \( m_{\text{susy}} \) is not too large. In fact, equating (17) and (15), one finds (\( \text{sgn}(\mu) = + \)) and

\[
m_{\text{susy}} \simeq \left( 62^{+4}_{-3} \text{ GeV} \right) \sqrt{\tan \beta} \tag{18}
\]
For $\tan \beta \sim 4$–$50$, those values are in keeping with mainstream SUSY expectations. So, supersymmetry is quite a natural explanation of the $a^\text{exp}_\mu$ deviation (if there really is one). Other ‘New Physics’ interpretations have been given. They include radiative mass models [6], extra dimensions [21] and many other possibilities. However, SUSY is so well motivated and so fundamental that it is by far the favoured theoretical explanation. If such an interpretation is correct, SUSY has a relatively low mass scale and will directly make its appearance at future high energy hadron–hadron and $e^+e^-$ colliders if not before. If that happens, the muon g-2 should indeed be heralded as a harbinger of supersymmetry [6].

4. Electroweak natural relations

In the 1970s, renormalization of the $SU(2) \times U(1)$, electroweak theory was proven, QCD was established, and the standard model emerged as a paradigm of elementary particle physics. Within that renormalizable framework, one could carry out perturbative calculations of observable quantities and compare those predictions with precise experimental measurements, much in the spirit of QED. However, electroweak theory has many more parameters than QED, making it more diverse and a richer testing ground for precision measurements. Furthermore, because weak interactions are naturally short-distance phenomena, they start at high mass scales ($m_w \simeq 80$ GeV) and are, therefore, better suited for probing ‘New Physics.’ Just as $a_\mu$ turned out to be $(m_\mu/m_e)^2 \simeq 40,000$ times more sensitive than $a_e$ to high scale physics, precision weak interaction studies are roughly $(m_w/m_\mu)^2 \simeq 10^6$ times more sensitive than $a_\mu$. Of course, weak interaction measurements cannot compete with the 1 part/billion studies of QED. However, they have reached $\pm 0.1\%$ level, making them powerful probes of ‘New Physics.’

Of particular importance for electroweak precision studies are the natural relations [22]

$$\sin^2 \theta_w^0 = \frac{G_\mu}{G^2} = 1 - \left(\frac{m_\mu}{m_\tau}\right)^2$$

(19)

that relate some of the fundamental bare parameters of the theory. Due to an underlying global $SU(2)_L$ symmetry, those relations continue to hold among renormalized parameters up to finite, calculable radiative corrections [8]. Because the parameters include couplings, masses and a mixing angle, the radiative corrections (as well as potential tree level deviations from equation (19)) are sensitive to a vast array of potential ‘New Physics’ effects. Indeed, the situation is quite different from pure QED where effectively one measures only $\alpha$ in different ways and compares them. Any ‘New Physics’ absorbed into $\alpha$ will not be revealed in such a procedure.

Radiative corrections to equation (19) will depend on the renormalized parameters compared. The usual prescription is to pick $\alpha$, $G_\mu$ and $m_\tau$ (pole mass) because they are so well known

$$\alpha^{-1} = 137.035\,998\,77(40)$$
$$G_\mu = 1.16637(1) \times 10^{-5}\,\text{GeV}^{-2}$$
$$m_\tau = 91.1875(21)\,\text{GeV}$$

(20)

and then compare with $m_w$ (pole mass) or $\sin^2 \theta_w(m_\tau)$ where the weak mixing angle is best measured at the Z pole. (For those who prefer the $\sin^2 \theta_w^\text{eff}$ definition, one has the simple translation $\sin^2 \theta_w^\text{eff} = \sin^2 \theta_w(m_\tau) + 0.00028$ [23].) To obtain the radiative corrections to those relations, one must compute loop corrections to: photon vacuum polarization, muon decay (used to derive $G_\mu$), Z and W self-energies, and whatever process is used to extract...
\[ \sin^2 \theta_w(m_z)_{\text{MT}} \]. Such calculations have been carried out, in some cases through two loops. They are embodied in the now famous radiative corrections functions \[ \Delta r(m_t, m_H, \Delta \alpha_{\text{Had}}) \]

\begin{equation}
\begin{aligned}
\Delta r(m_t, m_H, \Delta \alpha_{\text{Had}}) &= 1 - \frac{\pi \alpha}{\sqrt{2} G_{\mu} m_w^2} \left( 1 - \frac{m_w^2}{m_H^2} \right) \\
\Delta \hat{r}(m_t, m_H, \Delta \alpha_{\text{Had}}) &= 1 - \frac{2 \sqrt{2} \pi \alpha}{G_{\mu} m_z^2 \sin^2 2\theta_w(m_z) m_{\text{MT}}} 
\end{aligned}
\end{equation}

which are modern day analogs of \( g - \frac{3}{2} \) (For a more complete discussion of \( \Delta r \) and \( \Delta \hat{r} \), see the J J Sakurai lecture by Sirlin [24] in this volume.) Those functions contain important large orders of \( \left( \frac{\alpha}{\pi} \frac{m_t^2}{m_z^2} \right) \) and smaller orders of \( \left( \frac{\alpha}{\pi} \ln \frac{m_H}{m_w} \right) \) corrections as well as hadronic vacuum polarization contributions (from the renormalization \( \alpha \) [25]) which are denoted by \( \Delta \alpha_{\text{Had}} \) (leptonic vacuum polarization is also included by has little uncertainty; so, I do not display it as a functional variable). Those vacuum polarization effects are large, they give rise to a running \( \alpha(\mu) \) which starts at \( \alpha^{-1}(0) \simeq 137 \) and \( \alpha^{-1}(m_z) \simeq 1/129 \), a 6% shift [25]. Of course, if ‘New Physics’ is present in the quantum loops, it will also be present in \( \Delta r \) and \( \Delta \hat{r} \).

Using \( m_w \) or \( \sin^2 \theta_w(m_z)_{\text{MT}} \) as input in equation (21), the top quark mass [26] was well predicted \( (165 \sim 190 \text{ GeV}) \) before its discovery, a major success for precision measurements. Now the top quark mass is fairly well measured

\[ m_t \simeq 174.3 \pm 5.1 \text{ GeV} \]

so, we can concentrate on constraining the Higgs mass, although sensitivity to \( m_H \) is, unfortunately, only logarithmic. Of course, the \( m_H \) prediction will carry uncertainties from the input \( (m_w \) or \( \sin^2 \theta_w(m_z)_{\text{MT}} \) as well as \( m_t \) and \( \Delta \alpha_{\text{Had}} \). My discussion will focus mainly on \( \Delta \alpha_{\text{Had}} \) since it is strongly correlated with the primary uncertainty in \( a_{\mu}^{\text{SM}} \) which comes from \( a_{\mu}^{\text{Had}} \) (vac. pol).

As a reference for comparison consider

\[ \Delta \alpha_{\text{Had}}^{\text{SM}} = 0.02761 \]

a currently used central value. Based on that value, Sirlin has given [24] the following predictions (using \( \Delta r \) and \( \Delta \hat{r} \))

\begin{equation}
\begin{aligned}
m_H &= 23^{+49}_{-23} \text{ GeV, } <122 \text{ GeV(95% CL)} \quad \text{from } m_w = 80.451(33) \text{ GeV} \\

m_H &= 59^{+50}_{-20} \text{ GeV, } <158 \text{ GeV(95% CL)} \quad \text{from } \sin^2 \theta_w(m_z)_{\text{MT}} = 0.23085(21)
\end{aligned}
\end{equation}

where only the leptonic asymmetry determinations of \( \sin^2 \theta_w(m_z)_{\text{MT}} \) are used in equation (25). (Values of \( \sin^2 \theta_w(m_z)_{\text{MT}} \) from the \( z \rightarrow bb \) forward–backward asymmetry are larger; however, systematic effects make their interpretation more questionable. I do not consider them here, but note they are inconsistent with equation (25).) Global fits to all electroweak precision data give a higher prediction [27]

\begin{equation}
\begin{aligned}
m_H &= 85^{+54}_{-34} \text{ GeV, } <196 \text{ GeV(95% CL)} \\

m_H &\gtrsim 114 \text{ GeV}.
\end{aligned}
\end{equation}

Indeed, the values derived from \( m_w \) and \( \sin^2 \theta_w(m_z)_{\text{MT}} \) are noticeably on the lower side; but global fits wash out what might otherwise be interpreted as a deviation from the standard model which include lot of other data (after fits).
How will updated determinations of $\Delta \alpha^{\text{Had}}$ via $e^+e^- \rightarrow \text{hadrons}$ and $\tau \rightarrow \nu_\tau + \text{hadrons}$ affect these predictions. Remember, there is a significant $e^+e^-$ versus tau data discrepancy as manifested in a $\alpha^{\text{Had}}_{\text{vac. pol}}$. Scaling the results of Davier et al \cite{4} for $\Delta \alpha^{\text{Had}}$ and increasing the $\tau$ contribution (value as previously explained), I find (roughly)

$$\Delta \alpha^{\text{Had}} \simeq 0.02752 \, (\text{new } e^+e^- \text{ data})$$

$$\Delta \alpha^{\text{Had}} \simeq 0.02780 \, (\tau \text{ data})$$

which are to be compared with equation (23). The recent $e^+e^-$ data will increase the prediction of $m_H$ somewhat, slightly improving agreement with the experimental bound in equation (27). On the other hand tau data will lead to an even smaller prediction for $m_H$. For comparison with equations (24), (25) and (26), I give the predictions which correspond to the $\Delta \alpha^{\text{Had}}$ values in equations (28) and (29).

$$m_H = 24^{+50}_{-24} \, \text{GeV}, \, <125 \, \text{GeV} \, (95\% \, \text{CL}) \, (\text{new } e^+e^- \text{ data})$$

$$m_H = 21^{+46}_{-21} \, \text{GeV}, \, <115 \, \text{GeV} \, (95\% \, \text{CL}) \, (\tau \text{ data})$$

from $m_w = 80.451(33) \, \text{GeV}$

$$m_H = 63^{+53}_{-31} \, \text{GeV}, \, <170 \, \text{GeV} \, (95\% \, \text{CL}) \, (\text{new } e^+e^- \text{ data})$$

$$m_H = 51^{+46}_{-26} \, \text{GeV}, \, <139 \, \text{GeV} \, (95\% \, \text{CL}) \, (\tau \text{ data})$$

from $\sin^2 \theta_w(m_Z)m_{\tau} = 0.23085(21)$

$$m_H = 51^{+36}_{-24} \, \text{GeV}, \, <123 \, \text{GeV} \, (95\% \, \text{CL}) \, (\text{new } e^+e^- \text{ data})$$

$$m_H = 42^{+30}_{-20} \, \text{GeV}, \, <102 \, \text{GeV} \, (95\% \, \text{CL}) \, (\tau \text{ data})$$

average of (30) + (31).

In the case of the global fit in equation (26), the Higgs masses will shift down by roughly 12% for $\tau$ data and up by about +6% for the new $e^+e^-$ data.

What does all this mean? To me, it suggests that the 3 sigma deviation in $\mu - a_{\mu}^{\text{SM}}$ may be real, since trying to alleviate the difference by employing the larger hadronic vacuum polarization contribution suggested by tau data will significantly lower an already precariously low prediction for $m_H$. That effect is particularly pronounced when using $\sin^2 \theta_w(m_Z)m_{\tau}$. Indeed, trying to explain the entire difference in $a_{\mu}^{\exp} - a_{\mu}^{\text{SM}}$ via hadronic vacuum polarization would further lower the $m_H$ prediction to an unacceptable level in any fit. Of course, one could instead argue that ‘New Physics’ is entering $\Delta \alpha$ and $\Delta \tau$ rather than $a_{\mu}$. However, the most natural explanation \cite{28} in that case also seems to be light sleptons, sneutrinos and gauginos, which should impact $a_{\mu}$. So, in fact, supersymmetry may be causing the 3 sigma deviation in $a_{\mu}^{\exp} - a_{\mu}^{\text{SM}}$ as well as the tendency for $m_w$ and $\sin^2 \theta_w(m_Z)m_{\tau}$ to predict a low $m_H$. We may be seeing several small hints of supersymmetry starting to appear as quantum loop effects. (I should add that the particularly low prediction for $m_H$ from $m_w$ may also indicate that $m_t$ is larger than the 174.3 GeV central value usually assumed.)

It will be interesting to see if the hints of ‘New Physics’ gleamed above are real or merely in the eyes of this beholder.

5. Outlook

The recent $a_{\mu}^{\exp}$ result combined with precision measurements of $m_w$ and $\sin^2 \theta_w(m_Z)m_{\tau}$ are quite exciting if they are really starting to signal ‘New Physics.’ Soon we may have
confirmation or negation of the $e^+e^-\rightarrow \gamma + \text{hadrons}$. In addition, a relatively light Higgs as suggested by $m_w\sin^2\theta_w(m_z)$ and global fits may be close at hand with potential discovery at Fermilab [30]. Finding that the Higgs is light and that SUSY is showing up in $a_\mu$ would represent major successes for precision measurements.

In the longer term, future high luminosity $e^+e^-$ colliders might allow us to push measurements of $m_w$ and $\sin^2\theta_w(m_z)$ to $\pm 0.01\%$. Of course, to utilize such precision one would have to resolve the $e^+e^-\tau$ discrepancy in $\Delta\alpha_{\text{had}}$. Similarly, $a_\mu^{\text{exp}}$ could in principle be pushed another order of magnitude. In the hadronic vacuum polarization uncertainty can be overcome and the light by light hadronic loop calculation improved, such a measurement could be used (if SUSY is correct) to provide a very precise determination of $\tan \beta$.

Eventually, when ‘New Physics’ is directly uncovered at collider facilities, precision measurements of its properties will be carried out and used to search for even higher scale phenomena. The cycle will be repeated. Michelson would have been surprised, amazed and very pleased by the power of precision measurements.

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