University of California, Berkeley
Physics 8A Spring 2007 (*Yury Kolomensky*)

SOLUTIONS TO PRACTICE PROBLEMS FOR MIDTERM II

Maximum score: 100 points

1. *(20 points)* **Road Accident**
An H2 Hummer, traveling at a speed of 80 mph and weighing 9000 lbs, collides head-on with a Wolksvagen Beetle, weight 2000 lbs, which was obeying the 65 mph speed limit. Assuming that the collision was perfectly inelastic, how much energy was transformed into heat at impact ?

1. Solution

In the inelastic collision, the two objects "stick", and travel after collision with the same velocity V. Let v_1 and m_1 be the speed and mass of the Hummer, and v_2 and m_2 be the speed and momentum of the Wolksvagen. Momentum conservation implies

$$m_1 v_{1x} + m_2 v_{2x} = (m_1 + m_2)V$$

We'll choose the X axis along direction of $\vec{v}_1$. Since the collision is head-on, $v_{1x} = v_1$, and $v_{2x} = -v_2$. Hence,

$$V = \frac{m_1 v_1 - m_2 v_2}{m_1 + m_2} \quad (1)$$

The kinetic energy before the collision was

$$K_i = \frac{m_1 v_1^2}{2} + \frac{m_2 v_2^2}{2} \quad (2)$$

and after the collision it is

$$K_f = \frac{(m_1 + m_2)V^2}{2} \; . \quad (3)$$

Combining Equations (1-3) and a bit of algebra, we find the amount of mechanical energy lost, or transformed into heat:

$$\Delta E = K_i - K_f = \frac{m_1 m_2 (v_1 + v_2)^2}{2(m_1 + m_2)} \approx 1.5 \text{ MJ}$$

2. *(25 points)* **Grizzly Peak Rd**
On my daily way to Berkeley, I usually take the Grizzly Peak Road, avoiding traffic in the Caldecott Tunnel. Grizzly Peak Rd is a narrow two-lane road, with some nice hairpin turns and stunning views of the Bay (can't find a better start to a day ! But I digress). One of the turns on this road has a radius of about 10 meters.

(a) (*10 points*) Assuming the static coefficient of friction $\mu = 1$ between my tires and the pavement, what is the maximum safe speed for this turn ?

(b) (*15 points*) A motorcyclist, rounding the curve at the same speed, will have to tilt the bike into the turn to maintain balance. What angle with the horizontal does he make?

2. Solution

(a) The first part of this problem is straightforward. The car experiences three forces: gravity mg pointing downward, normal force N pointing upward, and static friction F_{fr} pointing toward the center of the turn. The balance of forces in the vertical direction implies

$$N = mg$$

The friction force provides centripetal acceleration:

$$F_{fr} = \frac{mv^2}{R}$$

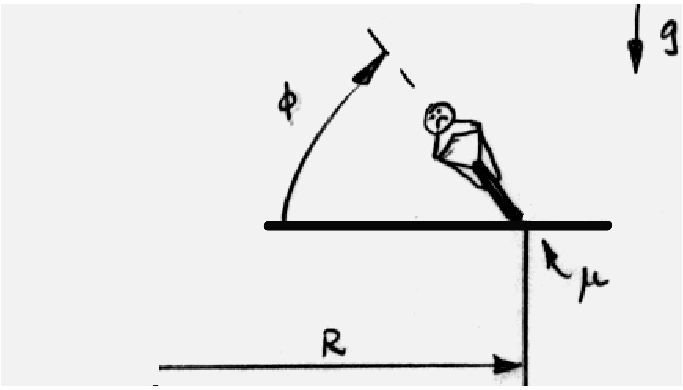
where m is the mass of the car, v is the linear velocity, and $R = 10$ m is the radius of the turn. At the largest possible velocity, friction force reaches its maximum possible value

$$F_{fr}^{\max} = \mu N = \mu m g$$

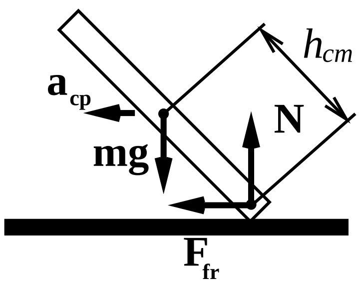
Therefore, the velocity has to be smaller than

$$v^{\max} = \sqrt{\frac{F_{fr}^{\max} R}{m}} = \sqrt{gR} = 10 \text{ m/s} = 22 \text{ mph}$$

(b) This part is a classic problem, and it describes a situation we all experience when we ride a bike, skate, etc. From experience, we know that in order to turn, the bike needs to be tilted into the turn. Why ?



Let's consider the process and its free-body diagram below.



There are three forces acting on the motorcycle: friction $\mathbf{F}_{fr}$, normal force $\mathbf{N}$ (both applied to the contact point with ground), and gravity $m\mathbf{g}$ (applied to the center of mass point). Let's assume that the distance between the CoM point and ground is h_{cm}. The force balance (Newton's law) equations are the same as in part (a):

$$N = mg\,;F_{fr} = ma_{cp} = m\frac{v^2}{R}$$

Now we need to write down the torque balance equations.

The choice of axis of rotation can greatly simplify this problem. The best choice here is the center of mass (CoM). Why ? If we choose CoM as the axis of rotation, we do not have to worry about torque due to gravity, and we do not have to worry about a_{cp}, the acceleration of the center of mass. This is a general property of the solid bodies. You can always represent its motion as a superposition of the motion of the center of mass (in this case, with acceleration a_{cp}), and the rotation *relative* to the CoM. In the torque equation about the CoM, only *real* forces will appear.

If the bike is in equilibrium, the net torque about CoM has to be zero. The equation is

$$F_{fr}h_{cm}\sin\phi - Nh_{cm}\cos\phi = 0$$

which, together with $F_{fr} = \mu N$ means

$$\tan\phi = \frac{N}{F_{fr}} = \frac{1}{\mu} = 1$$

or $\phi = 45°$.

You can also write the torque equation about the point of contact with ground, but then you have to take into account the fact that the center of mass is moving with acceleration a_{cp}. That means there is non-zero angular acceleration of the center of mass relative to the ground point, and that angular acceleration is created by the gravitational torque. The equation is

$$\tau = mgh_{cm}\cos\phi = (mh_{cm}^2)\alpha = (mh_{cm}^2)\frac{a_{cp}\sin\phi}{h_{cm}}$$

which simplifies to

$$\tan\phi = \frac{g}{a_{cp}}$$

Using $a_{cp} = F_{fr}/m = \mu g$, we get the same answer: $\phi = 45°$.

Finally, you can work out this problem using *fictitious* centrifugal force. We generally do not advise this, since the fictitious forces have to be added "by hand". But often they can greatly simplify the reasoning. In the reference frame of the bike, there is a fictitious centrifugal force, applied to the CoM point, and equal to $F_{cf} = -ma_{cp}$. This force applies an apparent torque on the body of

$$\tau_{cf} = F_{cf}h_{cm}\sin\phi$$

and this torque is balanced by the gravitational torque:

$$ma_{cp}h_{cm}\sin\phi = mgh_{cm}\cos\phi$$

which again leads to the same answer $\tan\phi = 1/\mu$.

The latter reasoning is in fact what you experience when you ride the bike. You have to tilt the bike into the turn in order to counteract the centrifugal torque, which is trying to push you in the direction away from the turn.

3. *(20 points)* **Kid on a Carousel**
A child of mass $m = 40$ kg stands on the edge of a rotating carousel (a uniform disk, rotating around a frictionless axle) of mass $M = 80$ kg. Initially, the carousel makes one revolution every 2 sec. The child makes her way to the middle of the carousel.

(a) (*15 points*) Calculate the final rate of revolution and the factor by which the kinetic energy of rotation has been increased.

(b) (*5 points*) Where does the increase in the rotational kinetic energy come from ?

3. Solution

(a) This problem demonstrates the conservation of angular momentum. The system containing the child and the carousel is closed, and if there is no external friction, the total angular momentum is conserved. The angular momentum of the rotating body is

$$L = I\omega$$

and the kinetic energy associated with rotation is

$$K = \frac{I\omega^2}{2}$$

where I is the moment of inertia of the system, and ω is the angular velocity. The moment of inertia of the system is the sum of the moment of inertia of the carousel

$$I_{car} = \frac{MR^2}{2}$$

and the moment of inertia of the child relative to the center of the carousel

$$I_{child} = mr^2$$

where r is the distance of the child to the center of the carousel. In the initial configuration,

$$L_0 = \left(\frac{MR^2}{2} + mR^2\right)\omega_0$$

where $\omega_0 = 2\pi/2 \text{ sec}^{-1} = \pi \text{ sec}^{-1}$ is the initial angular velocity. When the child is at the center, the angular momentum is

$$L = \frac{MR^2}{2}\omega \ .$$

Setting $L = L_0$, we can relate ω and ω_0:

$$\omega = \left(1 + \frac{2m}{M}\right)\omega_0$$

The angular velocity is related to the frequency (rate) of rotation f as $\omega = 2\pi f$, so

$$f = \left(1 + \frac{2m}{M}\right) f_0 = 1 \text{ sec}^{-1} = 1 \text{ Hz} \ .$$

We can also compute the fractional change in kinetic energy:

$$\frac{K}{K_0} = \frac{I\omega^2}{I_0\omega_0^2} = \frac{\omega}{\omega_0} = 2$$

(b) This should be obvious. Qualitatively, when the child is moving toward the center of the wheel, she is working against a fictitious centrifugal force. If she moves at constant velocity relative to the wheel, the centrifugal force is equalized by the force of friction between the child's feet and the wheel. The work done by this friction force increases the wheel's kinetic energy.

If you prefer, you can also look at this problem without fictitious forces. Look at the trajectory of the child relative to stationary ground: it is a spiral. To stay on this trajectory, there has to be a non-zero component of the friction force that points along this trajectory at every point. That component of the friction force does the work, which translates into the increase of the kinetic energy of the system.

4. *(15 points)* **Better Safe Than Sorry**

Before they learn how to swim, small children may use inflatable life jackets, or floaties (inflatable sleeves) to support them in water. A life jacket is supposed to support at least 15% of a child's body (head and shoulders) above water. You are writing a safety manual for a life jacket that has 1 gallon (3.8 L) volume when inflated. What is the maximum mass of a child that it can safely support ? Assume average density of a human body $\rho_h = 950 \text{ kg/m}^3$.

4. Solution

A fairly simple buoyancy problem. We need to find the volume of the life jacket such that the weight of the child is balanced by the buoyancy force. The buoyancy acts both on the child's body (the part immersed in water) and on the jacket. The child experiences the force

$$F_c = \rho_{H_2O} g V_{\text{submerged}}$$

where $V_{\text{submerged}} = 0.85 V_{\text{child}} = 0.85 m/\rho_h$ is the volume of the child in the water, and m is the mass of the child. The force on the jacket is

$$F_j = \rho_{H_2O} g V_j$$

where $V_j = 3.8$ L is the volume of the jacket. The force balance equation is

$$mg = \rho_{H_2O} g V_j + 0.85 mg \frac{\rho_{H_2O}}{\rho_h}$$

Solving it for the mass of the child, we find

$$m = \frac{\rho_{H_2O} V_j}{1 - 0.85 \rho_{H_2O}/\rho_h} = 36 \text{ kg}$$

This is a reasonable answer (36 kg $\approx$ 80 lb).

5. *(20 points)* **Municipal Water Supply**
Water pressure in the municipal water systems is usually maintained by water towers. Water tower is a large capacity water tank, elevated high above ground. You have probably seen many water towers in rural areas, where they stick out like a sore thumb (in the cities, they are often hidden on the roofs of tall buildings). A typical rural water tower could be $H = 50$ m (165 feet) high.

- **(a)** (*10 points*) Calculate the water pressure at ground level for such a tower.
- **(b)** (*10 points*) The water is delivered to houses through standard underground pipes of $d = 1 \text{ in} = 2.54 \text{ cm}$ diameter. What flow rate (volume per unit time) and water velocity could you expect in the second floor shower, 6 m above ground (if your neighbors are not using any water) ?

5. Solution
(a) This is a fairly straightforward application of the Torricelli's formula. The pressure of *non-moving* water at ground level is

$$p = \rho g H = 5 \text{ atm}$$

This is *gauge* pressure – since the water tower tanks are not typically evacuated, there is additional contribution from the atmospheric pressure (from the air above the water level in the tank). The total pressure is then be 6 atm.
(b) Now we are dealing with moving water, so will apply Bernoulli's equation. We will compare the pressure on the water tower tank (near the surface) and in the shower:

$$p_{atm} + \rho g H = p_{atm} + \rho g h + \frac{\rho v^2}{2}$$

where v is the velocity of water coming out of the pipe on he second floor of the house. Solving this equation for velocity, we find

$$v = \sqrt{2g(H - h)} = 30 \text{ m/s} .$$

The flow rate is simply

$$Q = vA = 15 \text{ L/s}$$