1. (10 points) **Blitz**
This is a set of simple questions to warm you up. The problem consists of five questions, 2 points each.

1. Before his untimely retirement this year, Al MacInnis used to possess one of the hardest slap-shots in the NHL. Imagine Al shooting the puck with a speed of 100 mph from a distance of 60 feet from the goal. How much time does it take for the puck to get to the net? An answer to one significant digit would be sufficient.

   \[ t = \frac{v}{L} \]

   \[ t = 0.4 \text{ s} \]

   Surprisingly, that is actually a lot of time. A typical human has a reaction time of about 0.1 s, and top hockey goal-tenders can be quicker than that. So a good NHL goalie (like San Jose Sharks’ Evgeni Nabokov) will almost always block a direct shot from the blue line (a bit over 60 feet from the goal), unless he is screened and can’t see.

2. Circle correct answer. What is a “particle”?
   (a) Any part of an atom
   (b) An object that can be represented as a mass at a single point in space
   (c) A part of a whole
   (d) An object that can be represented as a single point in time
   (e) All of the above

   \[ \text{Answer: (b)} \]

3. Circle correct answer. When a vector sum of all forces acting on a body is zero, the body
   (a) moves with increasing speed
   (b) moves with decreasing speed
   (c) moves with constant speed, or remains at rest
   (d) All of the above
   (e) None of the above

   \[ \text{Answer: (e)} \]

4. Circle correct answer. When a vector sum of all external forces acting on a system of particles is zero, the total momentum of the system
   (a) increases
   (b) decreases
   (c) is conserved
   (d) All of the above
   (e) None of the above

   \[ \text{Answer: (e)} \]

5. Hoover Dam is 726 ft (221 m) high. If its hydro-electric power plant were fully efficient (i.e. all of the mechanical energy of the water is transformed to electricity), how much energy would one kilogram of water falling from the top of the dam generate?

   \[ \Delta E = mgh \]

   \[ \Delta E = 2.2 \text{ kJ} \]

   This amount of energy is sufficient to warm up the same kilogram of water by only \( 0.5^\circ C = 1^\circ F \).

2. (20 points) **John Kruger and his parachute**
There is a scene in the movie *Eraser*, where the hero John Kruger (Arnold Schwarzenegger) is falling head first from an airplane, trying to catch up with a folded parachute. Let’s see if Hollywood got some basic physics right.

In the scene, Kruger drops the parachute first. We will assume, that the parachute is falling with the constant vertical velocity \( v_p = 20 \text{ m/s} \) (that’s roughly the terminal velocity for a folded parachute). \( \Delta t = 7 \text{ seconds} \), after dodging a few bullets from the bad guys, Kruger lets go of the plane, and falls head first
after the parachute. We will assume that the drag force acting on John is negligible.

(a) (10 points) At what altitude, relative to the plane, does John catch up with the parachute?

(b) (5 points) What is his velocity at that point?

(c) (5 points) If he continues to fall for another 20 seconds, while trying to put on the parachute, how much farther does he fall? Again, assume that drag is negligible.

2. Solution

(a) Let’s start the clock at \( t_0 = 0 \) when John starts falling and use \( y_0 = 0 \) to be the altitude of the plane. Then, the parachute starts falling at \( t_p = -\Delta t \). Let’s also follow the tradition and point the \( Y \) axis up (so that negative altitude means, as usual, that the object is below the plane). You may have chosen to point your \( Y \) axis downward, in which case all positions and velocities below would switch sign. The equations of motion for John and the parachute are

\[
\begin{align*}
y_J(t) &= -\frac{gt^2}{2} \\
y_p(t) &= -v_p(t + \Delta t)
\end{align*}
\]

(\text{notice signs}). We want to find such time \( t_c \) so that \( y_J(t_c) = y_p(t_c) \). The equation for \( t_c \) follows from Eqs. (1,2):

\[
\frac{g}{2}t_c^2 - v_p t_c - v_p \Delta t = 0
\]

which has one allowed (positive) solution

\[
t_c = \frac{v_p + \sqrt{v_p^2 + 2v_p g \Delta t}}{g} = 7.7 \text{ s}.
\]

At the instance \( t = t_c \), John would be at the elevation

\[
y_J(t_c) = \frac{gt_c^2}{2} = -290 \text{ m}.
\]

(b)

\[
v_J(t_c) = -gt_c = -77 \text{ m/s}.
\]

This velocity is high, but even at 80 m/s, the drag force acting on John is not significant, if he is falling headfirst. His cross sectional area is about \( A \approx 0.04 \text{ m}^2 \), and the drag force is approximately

\[
F_d \approx 0.25 (\text{kg/m}^3)A v_J^2 = 60 \text{ N}
\]

or about 5% of Schwarzenegger’s weight. So ignoring drag for Kruger is justified.

(c) Between \( t = t_c \) and \( t_2 = t_c + 20 \text{ sec} \), John and his parachute have fallen by

\[
\Delta y = -v_J(t_2)(t_2 - t_c) - \frac{g}{2}(t_2 - t_c)^2 = -3.5 \text{ km}
\]

So John’s final altitude at that time \( t_2 \) would be

\[
y_2 = y_J(t_c) + \Delta y = -\frac{gt^2}{2} = -3.8 \text{ km}.
\]

Now, how realistic is that? First off, in the movie John spent about 20 sec (instead of 8) to catch the parachute. I guess that adds to the dramatics, but it just can’t be right, unless someone added a lead brick to John’s parachute bag (which would make \( v_p \) significantly larger). Also, in 20 sec, John would have fallen by 2 km – and would likely have created a significant crater on the ground. The total fall distance of almost 4 km is also problematic. In the movie, the pilot can be heard saying “Descending to 3000 feet” 20 sec before John jumps. Indeed, when John opens the door, there is no significant pressure difference between the plane and the outside, the oxygen masks don’t appear, etc. This means the plane was probably flying below 8000 feet, which is only 2.4 km. Yet, when John opens the parachute, he appears to still be fairly high in the air – at least 3000 feet or so.

The final goof that amused me is the motion along plane’s path (which we ignore for this problem, but have to account for in real life). Indeed, in 7 seconds, the jet, which should have had a cruising speed of at least 200 km/h, would have traveled by about 400 meters. The parachute, on the other hand, would quickly reach the horizontal velocity of \( v_p \) (it can be seen flying backwards relative to the plane). When John jumps, he still has a horizontal velocity along the plane’s direction – so he is flying away from the chute. How can he ever catch up? Schwarzenegger may be the Eraser, but he’s no Superman...

3. (25 points) Best punter in football

Shane Lechler, who plays for the Oakland Raiders of the NFL, is considered one of the best punters in football. According to his career statistics, his longest punt (longest distance he ever kicked the ball) is 74 yards (66
3. Solution

(a) (5 points) At what angle over the horizon did Shane most likely kick the ball, when he produced his longest kick? Explain.

(b) (5 points) Estimate (2 significant digits) the speed of the ball when it left Shane’s foot.

(c) (5 points) How long was the ball in the air? (This is called hang time in football.)

(d) (10 points) Rather than punting for distance, it is often more important to kick the ball in such a way so that a player from your team (the gunner) arrives at the spot where the ball lands exactly at the time when it lands. Assume that the gunner starts running \( \Delta t_g = 1.5 \) seconds before the kick is made from \( \Delta x = 13.5 \) meters (15 yards) in front of the punter. The gunner is running at a constant (very respectable) speed of \( v_g = 9 \) m/s. At what angle over the horizon should Shane kick, so that the ball and the gunner arrive at the same spot downfield at the same time? What is the length of the kick (total distance traveled by the ball along horizontal direction)? Assume that Mr. Lechler always kicks the ball with the same speed (found in part (b)).

3. Solution

(a) You may know that the right answer is \( \theta = 45^\circ \), but you need to justify it here (e.g., if we had to take into account that Lechler punts from an elevation of about 1 m, we’d find that the optimal angle would be slightly less than \( 45^\circ \)). The derivation goes as follows. The distance traveled by the ball is

\[
L = vt_f \cos \theta
\]  

where \( v \) is the speed of the ball at the time of the kick, \( \theta \) is the angle of the kick above the horizon, and \( t_f \) is the total flight time. If the ball starts and stops at ground level, we know that it spends time \( t_f/2 \) on the upward trajectory, and and another \( t_f/2 \) falling down. Hence, it takes \( t_f/2 \) seconds to reduce its vertical velocity from \( v \sin \theta \) to zero. Therefore, we find the flight time through velocity equation:

\[
0 = v \sin \theta - g \frac{t_f}{2} \rightarrow t_f = \frac{2v \sin \theta}{g} \tag{4}
\]

Plugging that into Eq. (3), you get that the range is

\[
L = \frac{v^2}{g} \sin 2\theta \tag{5}
\]

It is maximum if \( \sin 2\theta = 1 \), or \( \theta = 45^\circ \).

You did not have to provide the full derivation on the exam, but to get full credit for this part, you should have at least stated the conditions under which \( \theta = 45^\circ \) is the right answer.

(b) From Eq. (5), the maximum distance for the kick is

\[
L_{\text{max}} = \frac{v^2}{g}
\]

Plugging in \( L_{\text{max}} = 66 \) m, we get

\[
v = \sqrt{L_{\text{max}}/g} = 25 \text{ m/s}.
\]

(c) We have already found hang time \( t_f \):

\[
t_f = \frac{2v \sin \theta}{g} = 3.4 \text{ s}
\]

for \( \theta = 45^\circ \).

(d) This situation is a bit more complicated that Problem 2. Let’s start the clock at \( t_0 = 0 \) when the ball is kicked. The gunner starts running from positive position \( \Delta x \) at time \( -\Delta t_g \) (since he started before the ball). The equation for the gunner’s \( x \) position is

\[
x_g(t) = \Delta x + v_g(t + \Delta t_g) = (\Delta x + v_g \Delta t_g) + v_g t
\]

The second form is a bit more familiar: at the time the ball is kicked, the gunner is ahead of the ball by \( (\Delta x + v_g \Delta t_g) = 27 \) m. The position of the ball as a function of time is

\[
x_b(t) = vt \cos \theta
\]

The flight time is still given by Eq. (4). We now require that \( x_g(t) = x_b(t) \), and solve for \( t \):

\[
\frac{2v^2}{g} \sin \theta \cos \theta - \frac{2vv_g}{g} \sin \theta - (\Delta x + v_g \Delta t_g) = 0
\]

Its solution is too complicated to be done analytically (my apologies), so you should do it numerically. There

meters). In the following, we will ignore air resistance, and pretend that Shane kicks from ground level (which does not make much difference in the calculations).
are two possible solutions: $\theta_1 \approx 20^\circ$, which results in the range $L_1 = 42$ m, and $\theta_2 \approx 50^\circ$, which results in the range $L_2 = 65$ m (so this is better for long-distance punting). You will receive full credit for either of these solutions (to within $\pm 10^\circ$), provided the initial equation is correct.

4. (25 points) Cross a river
A group of hikers is using a rope trolley to cross a river. One of them attaches one end of the rope to a tree on the far bank of the river. The rest of the group, after securing the other end of the rope tightly to a tree on the near bank, pull themselves across the river one by one, while being suspended on the rope with a harness. The distance between the trees is $L = 20$ m. See picture below.

(a) (10 points) When the heaviest of the hikers, a 100 kg gentleman, is exactly midway between the trees, the rope sags by $H = 1$ m. What is the tension $T$ of the rope?

(b) (10 points) Hopefully, you have found out that the tension $T$ is large. Clearly, a few people would have a hard time stretching the rope to such tension. So it is common to use a block-and-tackle device to pull the rope until tension reaches the required value $T$. Block-and-tackle (remember lecture demonstration?) is a set of pulleys, arranged as schematically shown below (two pulleys on the left and one end of the tackle rope are connected to the tree, the other two pulleys are connected to the rope across the river). Suppose a single hiker wants to be able to tighten the rope across the river to tension $T$ found in part (a), by pulling with force $F = 500$ N ($\approx 110$ lbs). How many pulleys does he need in the block-and-tackle device? (If you did not get the answer in part (a), express your answer to part (b) symbolically, in terms of rope tension $T$).

(c) (5 points) The rope obeys Hooke’s law, in that its tension depends on its length as

$$T = k(L - L_0)$$

where $k$ is a constant and $L_0 = 19$ m is the length of the unstretched rope. How much work is required to stretch the rope to tension $T$ from part (a)?

4. Solution
(a) The force diagram for the hiker is shown below.

\[ \begin{align*}
\vec{T}_1 &+ \vec{T}_2 + \vec{m}\vec{g} = 0 \\
\end{align*} \]

In equilibrium,

$$\vec{T}_1 + \vec{T}_2 + \vec{m}\vec{g} = 0$$

The tension forces forces are equal in magnitude $|\vec{T}_1| = |\vec{T}_2| \equiv T$ and balance out in the horizontal direction. In the vertical direction, the force equation is

$$|\vec{T}_1| \sin \theta + |\vec{T}_2| \sin \theta - mg = 0$$

where $m = 100$ kg is the mass of the hiker. This yields, for the magnitude of tension $T$

$$T = \frac{mg}{2\sin \theta}$$

The angle $\theta$ can be found from right triangle formed by two sides $H$ and $L/2$:

$$\tan \theta = \frac{2H}{L} = 0.1 \approx \sin \theta$$

The last (approximate) equality is good to 0.5%. Plugging the numbers in, the magnitude of the tension of the rope is

$$T = \frac{mgL}{4H} = 5kN.$$
(b) The difference in length between the straight rope and the rope deflected by \( H = 1 \text{ m} \) is only 0.5\%, so the tension in a straight rope is also approximately \( T \approx 5 \text{ kN} \), or about half a ton (that was explicitly stated in part (b)). A single person can’t pull the rope that tightly without a mechanical advantage device. So a block-and-tackle (either a dedicated pulley device, or rigged with carabiners) is often handy. The tension needs to be high so that the rope trolley bridge does not deflect too much under the weight of a person (see part (a)): trying to pull oneself along the rope up the steep incline is not fun! Incidentally, 5 kN is not too much tension for good carabiners (often hold up to 20 tons).

We discussed the block-and-tackle in class. When the block-and-tackle is hooked up to the rope as shown, the mechanical advantage is \( N \), the number of pulleys in the system (or twice the number of pulley pairs). That is, the movable pulleys are connected directly to the rope being tightened. If we pull with zero acceleration, the tension of the tackle (the thread between the pulleys) is equal to \( F \), the force applied by the person. At the same time, the force applied to the rope is \( 2F \) per movable pulley, since the tension force of the tackle on each side of each movable pulley is directed to the left. To provide the total force \( T \) on the rope, you need to pull with force

\[
F = \frac{T}{N},
\]

therefore, the number of pulleys is

\[
N = \frac{T}{F} = 10.
\]

(c) The force required to keep the rope at length \( x > L_0 \) is \( T(x) = k(x - L_0) \). The work required to stretch the rope from its unstretched length \( L_0 \) to \( L \) is

\[
W = \int_{L_0}^{L} T(x) \, dx
\]

Plugging in the formula for \( T(x) \), the work is

\[
W = \frac{k(L - L_0)^2}{2}
\]

which is, incidentally, the formula for the potential energy of the spring (since the stretched rope obeys Hooke’s law, it works exactly like a spring). When the rope is fully stretched, its tension is \( T(L) = k(L - L_0) = 5 \text{ kN} \), and we know that \( L - L_0 = 1 \text{ m} \). The work is therefore

\[
W = \frac{T(L - L_0)}{2} = 2.5 \text{ kJ}
\]

5. (20 points) **Keep your head on a swivel**

It is Game 7 of the 2000 NHL Eastern Conference Finals between Philadelphia Flyers and New Jersey Devils. Eric Lindros, a 244 lb (110 kg) forward for the Flyers, is skating with a puck into the New Jersey zone, keeping his head down (big mistake). His speed is \( v_L = 10 \text{ m/s} \). At the blue line, he is met by the Devils’ defenseman Scott Stevens, who weighs 220 lbs (100 kg). Just before the collision, Stevens accelerates to \( v_S = 11 \text{ m/s} \), and meets Lindros head-on, planting his shoulder squarely into poor Eric’s chest.

(a) (5 points) What is the total momentum \( P \) and total kinetic energy \( K \) of the players before the collision?

(b) (10 points) Assuming the collision is elastic (i.e. no bones were crushed), what is Eric Lindros’ velocity after the collision?

(c) (5 points) If the collision lasted \( \Delta t = 0.2 \text{ sec} \), estimate the average force experienced by Lindros, and average acceleration of his body during the collision.

5. **Solution**

This problem is based on a real incident, which was one of the deciding factors in tilting that Game 7 in New Jersey favor (they won, and went on to take the Stanley Cup from the Dallas Stars in the final).

(a) Momentum is a vector quantity. Let’s point \( X \) axis in the direction of Lindros’ initial velocity. With this choice, the \( X \) projection of Lindros’ velocity is \( v_{Lx} = v_L = 10 \text{ m/s} \), and the \( X \) projection of Stevens’ velocity is \( v_{Sx} = -v_S = -11 \text{ m/s} \). The Lindros’ momentum projection is \( p_{Lx} = m_L v_L \), and Stevens’ momentum projection is \( p_{Sx} = -m_S v_S \). The total momentum along \( X \) direction is

\[
P_{x}^{\text{tot}} = p_{Lx} + p_{Sx} = m_L v_L - m_S v_S = 0
\]

The numbers work out just right: this is the center-of-mass frame! So the calculations of the scattering process should be fairly simple mathematically.
The kinetic energy of the players is
\[ K_{\text{tot}} = \frac{m_L v_L^2}{2} + \frac{m_S v_S^2}{2} = 11.6 \text{ kJ} \quad (7) \]

(b) The collision is head-on, so the velocity of each player has changed direction. Let’s label Lindros’ velocity projection after collision \( v'_{Lx} = -v'_L \), and the velocity of Stevens after the collision is \( v'_{Sx} = +v'_S \) (\( v'_L \) and \( v'_S \) are speeds, or magnitudes of the player velocities). Since the collision is elastic, both total momentum and total kinetic energy are conserved:
\[ P_{x}^{\text{tot}} = 0 = -m_L v'_L + m_S v'_S \quad (8) \]
\[ K_{\text{tot}} = \frac{m_L v'_L^2}{2} + \frac{m_S v'_S^2}{2} \quad (9) \]

To solve this system of equations, we need to express \( v'_S \) in terms of \( v'_L \) using Eq. (8):
\[ v'_S = v'_L \frac{m_L}{m_S} \]
and then plug it into Eq. (9):
\[ K_{\text{tot}} = \frac{m_L v'_L^2}{2} \left( 1 + \frac{m_L}{m_S} \right) \quad (10) \]

We can now plug in \( K_{\text{tot}} \) from Eq. (7) and solve Eq. (10) for \( v'_L \). On the other hand, we can realize that Eq. (10) is very similar to the expression for kinetic energy in Eq. (7). That is, if we solved for Lindros’ speed after collision \( v'_L \) in terms of his speed before collision \( v_L \), we’d find that they are equal:
\[ v'_L = v_L \]

This means that his velocity is
\[ v'_L = -v_L = -10 \text{ m/s} \]

This is a very general result for the center of mass system: in any two-body elastic collision, the center-of-mass speed of each object before and after the collision is preserved !

(c) We can now find the force that Eric experienced during the collision from momentum-impulse relation:
\[ \Delta p_{Lx} = m_L v'_{Lx} - m_L v_{Lx} = -2m_L v_L = F_x \Delta t \]

The force is
\[ F_x = -\frac{2m_L v_L}{\Delta t} = -11kN \]

During the collision, Eric experienced acceleration of
\[ a_x = \frac{F_x}{m_L} = -100 \text{ m/s}^2 \]
or 10g ! That’s the acceleration top fighter pilots experience during acrobatic maneuvers. Not surprisingly, Lindros, Flyers’ best but concussion-prone forward, was knocked out of the game. By some accounts, he was unconscious before he fell down to the ice... But the hit was clean and legal. Hockey is the fastest team sport, and so collisions can be violent ! That’s why you can often hear hockey announcers use the phrase “Keep your head on a swivel!” – that is, look out, especially when the likes of Stevens are lurking around the blue line !