Sound

Prof. Yury Kolomensky
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What We Know About Waves

• So far, we’ve learned how to describe a single wave in a 1d elastic medium
  - Longitudinal and transverse waves
  - Special case: sine waves

\[ v = \frac{\omega}{k} \]

\[ k = \frac{2\pi}{\lambda} \]

\[ \omega = \frac{2\pi}{T} \]
Superposition of Waves

\[ y'(x,t) = y_1(x,t) + y_2(x,t) \]

- **Consequences:**
  - **Interference**
    - Constructive (add in phase)
    - Destructive (add out of phase)
  - **Beats**
    - Two sources with different frequencies
    - Lab this week!
  - **Standing waves**
    - Superposition of waves traveling in opposite directions in finite medium
Standing Waves

\[ y'(x,t) = \left[ 2y_m \sin kx \right] \cos \omega t \]

- Add two waves propagating in opposite directions
  - Result is a wave that appears fixed in space
The displacement of a standing wave is given by the equation:
\[ y'(x,t) = [2y_m \sin kx] \cos \omega t \]
The position dependant amplitude is equal to \( 2y_m \sin kx \)

**Nodes:** These are defined as positions where the standing wave amplitude vanishes. They occur when \( kx = n\pi \quad n = 0,1,2, \ldots \)
\[
\frac{2\pi}{\lambda} x = n\pi \rightarrow x_n = n \frac{\lambda}{2} \quad n = 0,1,2,\ldots
\]

**Antinodes:** These are defined as positions where the standing wave amplitude is maximum.
They occur when \( kx = \left( n + \frac{1}{2} \right)\pi \quad n = 0,1,2,\ldots \)
\[
\frac{2\pi}{\lambda} x = \left( n + \frac{1}{2} \right)\pi \rightarrow x'_n = \left( n + \frac{1}{2} \right) \frac{\lambda}{2} \quad n = 0,1,2,\ldots
\]

**Note 1:** The distance between adjacent nodes and antinodes is \( \lambda/2 \)

**Note 2:** The distance between a node and an adjacent antinode is \( \lambda/4 \)
Resonances occur when the resulting standing wave satisfies the boundary condition of the problem. These are that the Amplitude must be zero at point A and point B and arise from the fact that the string is clamped at both points and therefore cannot move.

The first resonance is shown in fig.a. The standing wave has two nodes at points A and B. Thus $L = \frac{\lambda_1}{2}$ $\rightarrow \lambda_1 = 2L$. The second standing wave is shown in fig.b. It has three nodes (two of them at A and B)

In this case $L = 2\left(\frac{\lambda}{2}\right) = \lambda \rightarrow \lambda_2 = L$

The third standing wave is shown in fig.c. It has four nodes (two of them at A and B)

In this case $L = 3\left(\frac{\lambda}{2}\right) = \lambda \rightarrow \lambda_3 = \frac{2}{3}L$ The general expression for the resonant wavelengths is: $\lambda_n = \frac{2L}{n}$ $n = 1, 2, 3, \ldots$ the resonant frequencies $f_n = \frac{v}{\lambda_n} = n \frac{v}{2L}$
Boundary Conditions

• (see blackboard)
Examples: Musical Instruments

• Guitar
• Organ
• Bell
Sound

- Most of the wave phenomena can be demonstrated with sound
- Sound waves: longitudinal waves in elastic materials
  - Can be solids or fluids, but we normally associate sound with propagation through air

\[ s(x, t) = s_m \cos(kx - \omega t) \]

\[ \Delta p(x, t) = \Delta p_m \sin(kx - \omega t) \]
3d Picture

• Radial propagation
  - Wavefront: locations of maxima
  - Ray direction: normal to wavefront
  - Mathematically:

\[ \Delta p(r, t) = A(r) \sin(kr - \omega t) \]

\[ A(r) = \frac{A_0}{r} \quad (\text{for spherical waves}) \]
**Speed of Sound**

- The most basic parameter describing sound wave propagation
  - Property of the medium

<table>
<thead>
<tr>
<th>Medium</th>
<th>Speed of Sound (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Air (0ºC, 1 atm)</td>
<td>331</td>
</tr>
<tr>
<td>Air (20ºC, 1 atm)</td>
<td>343</td>
</tr>
<tr>
<td>Helium (0ºC, 1 atm)</td>
<td>970</td>
</tr>
<tr>
<td>Water</td>
<td>1480</td>
</tr>
<tr>
<td>Granite</td>
<td>6000</td>
</tr>
<tr>
<td>Aluminum</td>
<td>6420</td>
</tr>
</tbody>
</table>

\[ v = \sqrt{\frac{B}{\rho}} \]

- Depends on elasticity and density of the material
  - where \( B \) is bulk modulus and \( \rho \) is density
Intensity

Consider a wave that is incident normally on a surface of area $A$. The wave transports energy. As a result power $P$ (energy per unit time) passes through $A$. We define at the wave intensity $I$ the ratio $P / A$

$$I = \frac{P}{A} \quad \text{SI units: } \text{W/m}^2$$

The intensity of a harmonic wave with displacement amplitude $s_m$ is given by:

$$I = \left( \frac{\rho v \omega^2}{2} \right) s_m^2 \quad \text{In terms of the pressure amplitude} \quad I = \left( \frac{1}{2\rho v} \right) \Delta p_m^2$$

Consider a point source S emitting a power $P$ in the form of sound waves of a particular frequency. The surrounding medium is isotropic so the waves spread uniformly. The corresponding wavefronts are spheres that have S as their center. The sound intensity at a distance $r$ from S is:

$$I = \frac{P}{4\pi r^2}$$

The intensity of a sound wave for a point sources is proportional to $\frac{1}{r^2}$
The Decibel (dB)

The auditory sensation in humans is proportional to the logarithm of the sound intensity $I$. This allows the ear to perceive a wide range of sound intensities. The threshold of hearing $I_o$ is defined as the lowest sound intensity that can be detected by the human ear. $I_o = 10^{-12}$ W/m$^2$

The sound level $\beta$ is defined in such a way as to mimic the response of the human ear. $\beta = 10 \log \left( \frac{I}{I_o} \right)$ $\beta$ is expressed in decibels (dB)

We can invert the equation above and express $I$ in terms of $\beta$ as:

$I = I_o \times 10^{(\beta/10)}$

**Note 1:** For $I = I_o$ we have: $\beta = 10 \log 1 = 0$

**Note 2:** $\beta$ increases by 10 decibels every time $I$ increases by a factor of 10

For example $\beta = 40$ dB corresponds to $I = 10^4 I_o$
# Typical Sound Intensities

<table>
<thead>
<tr>
<th>Source</th>
<th>Intensity (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Threshold of hearing</td>
<td>0</td>
</tr>
<tr>
<td>Rustling Leaves</td>
<td>10</td>
</tr>
<tr>
<td>Whisper</td>
<td>20</td>
</tr>
<tr>
<td>Mosquito</td>
<td>40</td>
</tr>
<tr>
<td>Normal conversation</td>
<td>60</td>
</tr>
<tr>
<td>Busy street traffic</td>
<td>70</td>
</tr>
<tr>
<td>Vacuum cleaner</td>
<td>80</td>
</tr>
<tr>
<td>Large orchestra</td>
<td>100</td>
</tr>
<tr>
<td>iPod/CD player at max level</td>
<td>100</td>
</tr>
<tr>
<td>Sharks playoff game !</td>
<td>110 (113 after Marleau’s goal)</td>
</tr>
<tr>
<td>Typical threshold of pain</td>
<td>130</td>
</tr>
<tr>
<td>Military jet takeoff</td>
<td>140</td>
</tr>
<tr>
<td>Instant perforation of eardrum</td>
<td>160</td>
</tr>
</tbody>
</table>
Sound Frequencies

• Human ear is built for large dynamic range rather than high precision
  - Audible range: *linear* frequencies $f \sim 20$ Hz…20 kHz
    - Below 20 Hz: infrasound (which dogs hate)
    - Above 20 Hz: ultrasound (which bats and dolphins love)

• Musical note scale: *logarithmic*
  - $f_n = f_0 \times 2^{n/12}$ where $n$ is the step number relative to the frequency $f_0$ (base of the octave)
    - This scale is *logarithmic*, because $\log_2(f_n) = \log_2(f_0) + n/12$
    - 12 frets/octave on a guitar, 12 keys on a piano
    - Example: $f(C_4) = 256$ Hz, $f(C_5) = 512$ Hz
Doppler Effect

\[ f' = f \frac{v + v_D}{v - v_S} \quad f' > f \]

\[ f' = f \frac{v - v_D}{v + v_S} \quad f' < f \]

\[ f' = f \frac{v - v_D}{v - v_S} \]

\[ f' = f \frac{v + v_D}{v - v_S} \]

\[ f' = f \frac{v \pm v_D}{v \pm v_S} \]