Problem 7

\[ \begin{align*}
\theta_1 &= 30^\circ \\
\theta_2 &= 50^\circ \\
\theta_3 &= 70^\circ \\
\theta_1 &= 5 \text{ m} \\
\theta_2 &= 8 \text{ m} \\
\theta_3 &= 12 \text{ m}
\end{align*} \]

\[ \begin{align*}
d_1 &= d_1 \cos \theta_1 \hat{i} + d_1 \sin \theta_1 \hat{j} \\
d_2 &= d_2 \left[ \cos (\theta_1 + 180^\circ \theta_2) \hat{i} + \sin (\theta_1 + \theta_2 - \theta_3) \hat{j} \right] \\
d_3 &= d_3 \left[ \cos (\theta_1 + 180^\circ - \theta_2) \hat{i} + \sin (180^\circ + \theta_1 - \theta_2 - \theta_3) \hat{j} \right] \\
\vec{d}_1 &= 5(\cos 30^\circ \hat{i} + \sin (30^\circ + 50^\circ) \hat{j}) \\
\vec{d}_2 &= 8(\cos (30^\circ + 180^\circ - 50^\circ) \hat{i} + \sin (30^\circ + 180^\circ - 50^\circ) \hat{j}) \\
\vec{d}_3 &= 12(\cos (30^\circ + 180^\circ - 50^\circ - 80^\circ) \hat{i} + \sin (30^\circ + 120^\circ - 50^\circ) \hat{j}) \\
\vec{d}_f &= \vec{d}_1 + \vec{d}_2 + \vec{d}_3 = (4.33 - 7.52 - 2.08) \hat{i} \\
&\quad + (2.5 + 2.74 - 11.8) \hat{j} = -5.27 \hat{i} - 6.58 \hat{j} \\
|\vec{d}_f| &= \sqrt{(-5.27)^2 + (-6.58)^2} = 8.43 \text{ m}
\end{align*} \]

a) What is the magnitude of the total displacement?

b) What is the angle of the total displacement?

\[ \theta = \tan^{-1}\left(\frac{\text{d}_y}{\text{d}_x}\right) = \tan^{-1}\left(\frac{-6.58}{-5.27}\right) = 51.3^\circ \text{ or } 231^\circ \]
Power \( = 1.5 \text{ MW} = 1.5 \times 10^6 \text{ W} \)
\( V_0 = 10 \text{ m/s} \quad V_i = 25 \text{ m/s} \quad t = 6 \text{ min} = 360 \text{ s} \)

a) Calculate the mass of the train.

\[
P_{\text{aug}} = \frac{W}{t}
\]

\[
W = k_f - k_i \quad \text{work energy theorem}
\]

\[
P_{\text{aug}} \cdot t = k_f - k_i = \frac{1}{2} m (V_i^2 - V_0^2)
\]

\[
m = \frac{2 P_{\text{aug}} \cdot t}{(V_i^2 - V_0^2)} = \frac{1}{2} (1.5 \times 10^6 \text{ W}) (360 \text{ sec}) = 2057142 \text{ kg}
\]

\[
M \approx 2.1 \times 10^6 \text{ kg}
\]

b) Find the speed of the train as a function of time.

\[
P t = \frac{1}{2} m (V_i^2 - V_0^2)
\]

\[
V(t) = \sqrt{\frac{P t}{m} + V_0^2}
\]

\[
V(t) = \sqrt{1.5 \times 10^6 t + 10^2}
\]

V(t) = \sqrt{1.5 t + 100}

\[
f(t) = \frac{P}{V(t)} = \frac{1.5 \times 10^6 \text{ W}}{\sqrt{1.5 t + 100}} = \frac{1}{2} (1.5 t + 100)^{3/2}
\]

\[
f(t) = \frac{1}{2} (1.5 t + 100)^{3/2}
\]

\[
X = \int V(t) \, dt = \int_{10}^{120} (1.5 t + 100)^{1/2} \, dt = \frac{1}{2} (1.5 t + 100)^{3/2}
\]

\[
X = 6.7 \times 10^8 \text{ m}
\]
Chapter 8, #88 | Problem 3

\[ H = 850 \text{ m} \]
\[ h = 750 \text{ m} \]
\[ \theta = 30^\circ \]

a) What speed will the skier be going at the top of the lower peak? (No friction)

- Initial: \( U_i = mgh \)
- Final: \( U_f = mgh \)

Initially \( k_i = 0 \) (no velocity)

Finally \( k_f = \frac{1}{2} m v^2 \) (at top of lower peak)

\[ U_i + k_i = U_f + k_f \]
\[ mgh = mgh + \frac{1}{2} mv^2 \]
\[ v^2 = 2g(H-h) \]
\[ v = \sqrt{2g(H-h)} = \sqrt{2(9.8 \text{ m/s}^2)(850 \text{ m} - 750 \text{ m})} = 44 \text{ m/s} \]

b) What force \( \mu_k \) would make him stop at top of lower peak?

\[ F_N = F_g \cos \theta \] (Newton's law)
\[ F_k = \mu_k F_N = \mu_k F_g \cos \theta \]

\[ \Delta E_{th} = \int F_k \text{d}l = \mu_k F_g \cos \theta d \]

\( d = \) total distance along path, \( h = 3.2 \text{ km} = 3200 \text{ m} \)

at top of the peak, \( k_f = 0 \)

\[ U_i = U_f + \Delta E_{th} \]
\[ mgh = mgh + \mu_k F_g \cos \theta d \]
\[ mg(h-h) = \mu_k mg \cos \theta \]

\[ \frac{H-h}{\cos \theta} = \mu_k = \frac{850 \text{ m} - 750 \text{ m}}{3200 \text{ m} \cdot \cos(30^\circ)} = \frac{0.036}{\mu_k} \]
Problem 4

![Diagram of a ball moving on ice with forces labeled]

At what height does the ball lose contact with the ice?

\[ V_0 = 0 \]
\[ R = 13.8 \text{m} \]

The ice is frictionless.

Net force is towards center of the sphere.
\[ \ddot{a} = \frac{V^2}{R} \]

\[ \frac{mv^2}{R} = mg \cos \theta - F_N \]

When the boy leaves the ice \( F_N = 0 \)

\[ \frac{mv^2}{R} = mg \cos \theta \]

at the top of the ice \( U = 0 \) and \( K = 0 \)

When he falls off the ice
\[ U = -mgh = mg(R - R \cos \theta) = -mgR(1 - \cos \theta) \]

\[ K_f = \frac{1}{2}mv^2 \]

conservation of energy
\[ C = \frac{1}{2}mv^2 - mgR(1 - \cos \theta) \]

\[ \frac{1}{2}mv^2 = mgR (1 - \cos \theta) \]

Now substitute into above equation
\[ \frac{m}{R} \frac{v^2}{2} = \frac{m}{R} (2gR(1 - \cos \theta)) = mg \cos \theta \]

\[ 2(1 - \cos \theta) = \cos \theta \]

\[ 2 - 2 \cos \theta = \cos \theta \]

\[ 2 = 3 \cos \theta \]

\[ \cos \theta = \frac{2}{3} \]

Says: \( h = R \cos \theta = (13.8 \text{m})(\frac{2}{3}) = \boxed{9.2 \text{m} = h} \)
Problem 5

\( \theta = 30^\circ \)

\( \mu_k = 0.1 \)

\[ \begin{align*}
\text{a)} & \quad \text{assume } m_1 = 5 \text{ kg} \\
& \quad \text{What is the Acceleration?} \\
\text{b)} & \quad \text{assume } m_1 = 2 \text{ kg} \\
& \quad \text{What is the Acceleration?}
\end{align*} \]

\[ \begin{align*}
\Sigma F_2 &= m_2 \ddot{a} = m_2 g - \frac{\tau}{l} \\
\ddot{a} &= m_2 (g - \frac{\tau}{l}) \\
\end{align*} \]

\[ \begin{align*}
\Sigma F_1 &= m_1 \ddot{a} = \tau - m_1 g \sin \theta - F_k \\
\ddot{a} &= \frac{\tau}{m_1 l} - m_1 g \sin \theta - \mu_k m_1 g \cos \theta \\
\ddot{a} &= \frac{m_2 g - m_1 g \sin \theta - \mu_k m_1 g \cos \theta}{m_1 + m_2} \\
\end{align*} \]

\[ \ddot{a} = 2.0 \text{ m/s}^2 \]

b) For \( m_1 = 2 \text{ kg} \) using the same formula

\[ \ddot{a} = \frac{(5 \text{ kg} \cdot 9.81 \text{ m/s}^2 - 2 \text{ kg} \cdot 9.81 \text{ m/s}^2 \cdot 0.1 \cdot 2 \text{ kg} \cdot 9.81 \text{ m/s}^2 \cdot \cos 30^\circ)}{5 \text{ kg} + 2 \text{ kg}} \]

\[ \ddot{a} = 5.4 \text{ m/s}^2 \]

In both cases, the acceleration is up the plane and positive.
\[ \Sigma F_2 = m_2 \ddot{a} = \vec{T}_{T_2} \]
\[ \Sigma F, = m_1 \ddot{a} = \vec{T} + \vec{F}_{app} - \vec{F}_{T_1} - \vec{F}_{T_1} \]
\[ \Sigma F_{cord} = m_{cord} \ddot{a} = \vec{F}_{app} - \vec{F}_{T_2} \]

a) \[ m_1 \ddot{a} = \vec{F}_{app} - \vec{F}_{T_1} \quad \Rightarrow \quad \vec{F}_{T_1} = \vec{F}_{app} - m_1 \ddot{a} \]
\[ m_{cord} \ddot{a} = \vec{F}_{T_2} - \vec{F}_{app} \quad \Rightarrow \quad \vec{F}_{T_2} = \vec{F}_{app} - m_{cord} \ddot{a} \]
\[ \vec{F}_{T_1} = \frac{\vec{F}_{app} + \vec{F}_{T_2}}{2} \]

\[ \vec{F}_{app} = m_1 \ddot{a} = m_{cord} \ddot{a} + m_2 \ddot{a} \]
\[ \ddot{a} = \frac{\vec{F}_{app}}{m_1 + m_{cord} + m_2} = 1.7 m/s^2 \]

b) \[ m_1 \ddot{a} = \vec{F}_{app} - \vec{F}_{T_1} \quad \Rightarrow \quad \vec{F}_{T_1} = \vec{F}_{app} - m_1 \ddot{a} \]
\[ \vec{F}_{T_2} = 47N - 10kN(1.7m/s^2) = 0.8N \]
\[ \vec{F}_{T_1} = (12 kN)(1.7 m^2/s^2) = 21 N \approx \vec{F}_{T_2} \]
a) Describe motion in terms of kinetic and potential energy.

When the ball is released at t=0, the ball is at rest so \( y = 0 \). As the ball falls, potential energy decreases and kinetic energy increases. At the lowest point of the circular arc, potential energy equals zero and kinetic energy is at its maximum. At the opposite side, potential energy is maximum and kinetic energy equals zero.

b) Determine the speed of the bob as a function of position \( y \) as it swings back and forth.

\[
E_{\text{mech}} = U + V = \frac{1}{2} m v^2 + m g y
\]

At the point of release, \( y = 0 \):

\[
E_1 = m g y_0 = m g (l - l \cos \theta) = m g l (1 - \cos \theta)
\]

as \( v = 0 \).

At any other point along the swing:

\[
E_2 = \frac{1}{2} m v^2 + m g y
\]

Conservation of energy:

\[
E_1 = E_2
\]

\[
m g y_0 = \frac{1}{2} m v^2 + m g y
\]

\[
g y_0 = \frac{1}{2} v^2 + g y \rightarrow v = \sqrt{2 g (y_0 - y)}
\]

Using \( y = l - l \cos \theta \); \( y_0 = l - l \cos \theta_0 \):

\[
v = \sqrt{2 g l (\cos \theta - \cos \theta_0)}
\]
c) Speed at the lowest point of the swing?

\[ v = \sqrt{2gy_0} = \sqrt{2g \cdot 0.1 - \cos \theta} = v \]

d) Tension in the cord?

Using Newton's 2nd law in the radial direction

\[ \sum F = m \frac{v^2}{r} = T - mg \cos \theta \]

\[ T = m \frac{v^2}{r} + mg \cos \theta \]

Using \( v^2 = 2g \cdot (\cos \theta - \cos \theta_0) \) from part b,

\[ T = m \left( \frac{v^2}{r} + g \cos \theta \right) \]

\[ = 2mg(\cos \theta - \cos \theta_0) + mg \cos \theta \]

\[ = 3mg \cos \theta - 2mg \cos \theta_0 \]

\[ T = mg(3 \cos \theta - 2 \cos \theta_0) \]
A NCLP boat traveling at a speed of 2.4 m/s shuts off its engines at time $t=0$. How far does it travel before coming to rest if it is noted that after $3s$, its speed has dropped to half its original value? Assume that the drag force of the water is proportional to $v$.

The only force on the boat is the drag force: $F_d = -bv = ma = m\frac{dv}{dt}$

$-bv = m\frac{dv}{dt}$

$-\frac{b}{m} = \frac{1}{V} \frac{dv}{dt}$

$-\frac{b}{m} t = \ln \left( \frac{V}{V_0} \right)$

$V = V_0 e^{-\frac{bt}{m}}$

Using: $V = 2V_0$ and $t = 3s$

$\ln \left( \frac{1}{2} \right) = -\frac{b}{m} (3s) \Rightarrow \frac{b}{m} = 0.231$

Using $v = \frac{dx}{dt}$

$\int \frac{dx}{v} = \int V_0 e^{-\frac{bt}{m}} dt$

$x = V_0 \left( -\frac{m}{b} \right) \left( e^{-\frac{bt}{m}} - 1 \right)$

$e^{0} = 1$

$= \frac{Vo m}{b} \left( 1 - e^{-\frac{bt}{m}} \right)$

$x = \frac{(2.4m)}{(0.231)} \left( 1 - e^{-\frac{(0.231)(3s)}{0.231}} \right) = [5.19m = x]$
A motor cycle traveling at a speed of 95.0 km/h approaches a car traveling in the same direction at 75 km/h. When the motor cycle is 60.0 m behind the car, the rider accelerates uniformly and passes the car 10.0 s later. What was the acceleration of the motor cycle?

\[ V_{mc} = \frac{95 \text{ km}}{\text{h}} = 26.4 \text{ m/s} \]
\[ V_{cc} = \frac{75 \text{ km}}{\text{h}} = 20.8 \text{ m/s} \]
\[ X_{mc} = 0 \]
\[ X_{cc} = 60 \text{ m} \]
\[ t = 10 \text{ s} \]

Motorcycle

\[ V = V_{0mc} + at \]
\[ X - X_{mc} = V_{0mc}t + \frac{1}{2} at^2 \]
\[ X = V_{0mc}t + \frac{1}{2} at^2 \]

Now solve for \( a \):

\[ \frac{1}{2} at^2 = V_{cc}t + X_{cc} - V_{mc}t \]
\[ a = \frac{2}{t^2} \left( V_{cc}t + X_{cc} - V_{mc}t \right) \]
\[ a = \frac{2}{(10)^2} \left( 20.8 \text{ m/s} \cdot 10 \text{ s} + 60 \text{ m} - 26.4 \text{ m/s} \cdot \frac{26.4 \text{ m/s} \cdot 10 \text{ s}}{2} \right) \]
\[ a = 0.09 \text{ m/s}^2 \]
At serve, a tennis player aims to hit the ball horizontally. a) What minimum speed is required for the ball to clear the 0.9-m-high net about 15 m from the server if the ball is "launched" from a height of 2.5 m? b) Where will the ball land if it just clears the net (and will it be "good" in the sense that it lands within 7.0 m)? c) How long will it be in the air?

\[ x = x_0 + \frac{v_0 x}{t_0} \]

\[ t = \frac{x - x_0}{v_0} \]

\[ 0 = \frac{x - x_0}{v_0} \]

\[ t_0 = \frac{v_0}{g} \]

\[ x = \frac{v_0^2}{g} \]

\[ v_y = -\frac{1}{2} g t_0 \]

\[ v_x = \frac{x}{t_0} \]

\[ v_y = -\frac{1}{2} g \frac{x}{v_0} \]

\[ x = \frac{v_0^2}{g} \]

\[ \frac{v_y}{v_0} = \frac{\sqrt{v_y^2 + v_x^2}}{v_0} \]

\[ \frac{v_y}{v_0} = \frac{\sqrt{(v_y)^2 + (v_x)^2}}{v_0} \]

\[ x = \frac{\sqrt{v_y^2 + v_x^2}}{g} = \frac{\sqrt{(0.5 m/s)^2 + (\sqrt{\frac{v_0^2}{g}})^2}}{g} \]

\[ x = \frac{\sqrt{\frac{v_0^2}{g}}}{9} = \frac{\sqrt{\frac{(2.5 m)^2}{(0.3 m/s)^2}}}{9} \]

The ball is not "good".